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## Backtracking search optimization based economic environmental power dispatch problems

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### ABSTRACT

This paper presents the solution for a nonlinear constrained multi objective of the economic and emission load dispatch (EELD) problem of thermal generators of power systems by means of the backtracking search optimization technique. Emission substance like NO<sub>x</sub>, power demand equality constraint and operating limit constraint are considered here. The aim of backtracking search optimization (BSA) is to find a global solution under the influence of two new crossover and mutation operations. BSA has capability to deal with multimodal problems due to its powerful exploration and exploitation capability. BSA is out of excessive sensitivity to control parameters as it has single control parameter. The performance of BSA is compared with existing newly developed optimization techniques in terms quality of solution obtained, computational efficiency and robustness for multi objective problems.

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### Introduction

The traditional economic load dispatch (ELD) problem is an optimization problem looks for the best available generation units in order to provide the most efficient, reliable and low cost of generation while satisfying several equality and inequality constraints. To minimize the fuel cost of economic dispatch is inadequate when environmental emissions are also to be included in the operation of different power plants. Due to increasing awareness of environmental protection since 1970s with implementation of several pollution control acts, power plant are bounded to considered emissions like NO<sub>x</sub>, SO<sub>x</sub>, CO<sub>x</sub>, etc. in the ELD of power system studies to achieve minimum levels of pollution with the cheapest energy. The Economic emission dispatch (EED) problem plays an important role in the optimal amount of the generated power for the fossil-based generating units in the system by minimizing the emission level. However, EED problems cannot be handled by conventional single objective optimization techniques. Thus, the concept of economic emission load dispatch (EELD) has been implemented to figure out into a nonlinear multi objective optimization problem by considering both the objective of minimum cost of generation and as well as minimum emission level at the same time with heavy equality and inequality constraints.

In 1986 the first influence of power pools to solve the EED problem by considering emission as a single-objective optimization was described in [1]. Several approaches have been proposed as a multiple-objective optimization problem to minimize the total cost of generation and pollution control simultaneously in [2,3]. EL-Keib et al. applied air pollution act in economic dispatch problem in [4]. Nanda et al. used classical based techniques in EELD as a multiple-objective optimization problem to minimize the total cost of generation and pollution control simultaneously [5]. Economy, security and environment protection had been discussed in [6]. A linear programming technique was also applied in multi-objective economic load dispatch problem in [7] where single-objective is considered one at a time. Dhillon et al. and Chang et al. both used the cost of generation and emission accordingly as a single objective in [8,9].

Real-world power system optimization problems are often very complicated because of their high complexity and fuzziness. The conventional methods were applied to solve EELD problems. In case of conventional method, the essential assumption is that, the incremental cost of the generating units are monotonically increasing or piece-wise linear. As the effect of valve point loading is included in the mathematical problem formulation of practical EELD problem with a sinusoidal term along with normal quadratic fuel cost function or quadratic emission function, therefore modified fuel cost function or quadratic emission function become purely non linear and the resulting EELD problems become totally non-convex optimization problem. Therefore, classical optimization techniques

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which solves this type of EELD problems, can give only approximate solution.

Therefore, presently soft computing techniques are applied to solve practical EELD problems and it outperforms other previously developed techniques due to following reasons:

1. It can use the previous knowledge for the solution of a problem and its behavior under various circumstances while finding new solutions.
2. It utilizes a population of points (potential solutions) in their search leading to parallel processing.
3. It uses direct fitness information instead of function derivatives or other related knowledge.
4. It mainly uses probabilistic rather than deterministic transition rules.

Abido [10–12] solved the multi-objective environmental and economic dispatch using non-dominated sorting genetic algorithm (NSGA) and evolutionary programming. The quadratic programming solution had been implemented in emission and economic dispatch problems in [13]. Srinivasan et al. solved the Multi-objective generation scheduling [14] by using a fuzzy optimal search technique. However, due to some disadvantages for using these methods the global optimum solution could not be achieved properly. Huang et al. proposed a new technique fuzzy satisfaction-maximizing decision approach [15] in Bi-objective power dispatch to overcome the problems observed in [14]. Yalcinoz et al. proposed a genetic algorithm with arithmetic crossover technique and multi-objective optimization method is used for EELD problem in [16,17]. Srinivasan et al. applied an evolutionary algorithm [18] based method to solve EELD problems. Multi-objective stochastic search technique was integrated in [19] to evaluate economic load dispatch problems. The weakness of this approach is enormous time-consuming. Fonseca et al. applied evolutionary algorithm in [20] to solve economic and emission load dispatch by considering either emissions as constraints for first objective function or cost of generation as a second objective function of a multi-objective optimization problem. In [21], Wei et al. discussed minimization of carbon oxide as an emission dispatch for better solution. AlRashidi et al. and Thakur et al. both tried to provide better solution by applying population based algorithm called particle swarm optimization (PSO) technique [22,23] in EELD problem. Perez-Guerrero RE et al. [24] applied another new population based technique differential evolution method to evaluate economic and emission load dispatch by considering either emissions as constraints or cost of generation as a second objective function of a multi-objective optimization problem. Wu et al. [25] used a multi-objective differential evolution (MODE) algorithm to solve EELD problem taking three multi-objectives of fuel cost, emission and system loss. Abou El Ela et al. [26] applied also the differential evolution algorithm to solve emission constrained economic power dispatch problem. A new hybrid bacterial foraging with PSO-DE algorithm was implemented to solve dynamic economic dispatch problem with security constraints in [27]. Hota et al. [28] incorporated also a new fuzzy based bacterial foraging algorithm (MBFA) to solve both single and multi-objective EELD problems. In 2008, Biogeography-Based Optimization (BBO) algorithm developed by Dan Simon, proved it's advantage to solve different optimization problems. In 2010, A. Bhattacharya et al. applied BBO successfully to incorporate it to various multi-objective EELD problems in [29]. The above-mentioned technique has comparatively fast, reasonable nearly global optimal solution with other soft computing techniques. Hybrid technique of differential evolution and biogeography-based optimization (DE/BBO) [30] has been employed to solve different EELD problem in search for much improved and fast output, compared to those of

individual techniques. Recently Rajasomashekar et al. proposed BBO algorithm to find out a new approach to find out the best compromising solution between fuel cost and NO<sub>x</sub> emission in EELD problems [31].

Niknam et al. used two different efficient evolutionary techniques known as new adaptive particle swarm optimization [32] and Modified Shuffle Frog Leaping Algorithm with Chaotic Local Search [33] to solve Non-smooth economic dispatch problem. Celal Yasar et al. [34] applied genetic algorithm integrated with conic scalarization method to convert multi-objective problem into single objective problem and solved the emission dispatch problem of power system. Again the authors applied combined modified subgradient technique integrated with harmony search [35] to solve economic dispatch problems. Chatterjee et al. [36] and Shaw et al. [37] initiated an opposition based learning scheme within basic Harmony Search Algorithm and gravitational search algorithm to solve combined economic and emission load dispatch problems.

The major drawbacks of Evolutionary algorithms, swarm intelligence and many others population based and bio-inspired algorithm are complicated computation, using lots of parameters. For that reason these algorithms are not user-friendly for beginners. Moreover, the optimization methodologies which have been developed to solve EELD problem, the complexity of the task reveals the necessity for development of efficient algorithms to locate the optimum solution accurately and computationally efficient way.

In recent times, a new optimization technique based on the concept of three new operators- selection, mutation and crossover, called backtracking search optimization algorithm (BSA) has been proposed by Pinar Civicioglu [38]. BSA has a random mutation strategy that uses only one direction individual for each target individual. In mutation operation the algorithm can control the amplitude of the search direction in a very balanced and efficient manner. During mutation it can generate numerically large amplitude values which in turn help to find solutions far way from its present state and make the algorithm suitable for a global search. At the same time it can generate small amplitude values which in turn help to find solutions in neighborhood of its present state and make the algorithm suitable for a local search. Apart from this, BSA possesses a memory in which it stores a population from a randomly chosen previous generation for use in generating the search- direction matrix for next iteration. Thus, BSA's memory allows it to take advantage of experiences gained from previous generations when it generates a trial preparation. The historical population used in selection operation helps for the calculation of the search-direction values for a randomly selected solution of previous generation, to generate more efficient trial individual. Crossover strategy uses non-uniform and complex structure that ensures creation of new trial individuals in each generation. Boundary control mechanism is effectively used for achieving population diversity. Due to the attractive and versatile qualities of BSA, it has been observed that the performance of the algorithm is quite satisfactory when applied to solve continuous benchmark optimization problems [38].

The all-around qualities and improved performance of BSA to solve different optimization problems has motivated the present authors to implement this newly developed algorithm to solve a basic but complex power system optimization problem e.g. ELD, to realize its future scope in the field of power system optimization.

Section 'Mathematical formulation of EELD problems' of the paper provides a brief mathematical formulation of different types of EELD problems. The concept of Backtracking Search algorithm is described in Section 'Backtracking search optimization (BSA)' short description of the BSA algorithm and it used in EELD problems. Simulation studies are discussed in Section 'Numerical examples

and simulation results'. The conclusion is drawn in Section 'Conclusion'

### Mathematical formulation of EELD problems

The following objectives and constraints are considered for EELD problem.

#### Economic load dispatch

The fuel cost function  $F_1$  of economic load dispatch problem is presented as given below

$$F_1 = \left( \sum_{i=1}^N F_i(P_i) \right) = \left( \sum_{i=1}^N a_i + b_i P_i + c_i P_i^2 + |e_i \times \sin\{f_i \times (P_{i\min} - P_i)\}| \right) \text{ \$/h} \quad (1)$$

where  $F_i(P_i)$  is the  $i$ th generator cost function for  $P_i$  output;  $a_i$ ,  $b_i$  and  $c_i$  are the  $i$ th generator's cost coefficients;  $N$  is the number of generators. The objective function of (1) is minimized subject to following constraints:

#### Real power balance constraint

$$\sum_{i=1}^N P_i - (P_D + P_L) = 0 \quad (2)$$

The total transmission network losses  $P_L$  can be expressed using B-coefficients as given below

$$P_L = \sum_{i=1}^N \sum_{j=1}^N P_i B_{ij} P_j + \sum_{i=1}^N B_{0i} P_i + B_{00} \quad (3)$$

#### Generator capacity constraints

From each unit power  $P_i$  generated shall be within their lower limit  $P_i^{\min}$  or upper limit  $P_i^{\max}$ . So that

$$P_i^{\min} \leq P_i \leq P_i^{\max} \quad (4)$$

The power level of  $N$ th generator (i.e. Slack Generator) is given by the following equation

$$P_N = P_D + P_L - \sum_{i=1}^{(N-1)} P_i \quad (5)$$

The transmission loss  $P_L$  is a function of all the generators including that of the slack generator ( $N$ th Generator) and it is given by

$$P_L = \sum_{i=1}^{N-1} \sum_{j=1}^{N-1} P_i B_{ij} P_j + 2P_N \left( \sum_{i=1}^{N-1} B_{Ni} P_i \right) + B_{NN} P_N^2 + \sum_{i=1}^{N-1} B_{0i} P_i + B_{0N} P_N + B_{00} \quad (6)$$

Expanding and rearranging, Eq. (5) using (6) becomes

$$B_{NN} P_N^2 + \left( 2 \sum_{i=1}^{N-1} B_{Ni} P_i + B_{0N} - 1 \right) P_N + \left( P_D + \sum_{i=1}^{N-1} \sum_{j=1}^{N-1} P_i B_{ij} P_j + \sum_{i=1}^{N-1} B_{0i} P_i - \sum_{i=1}^{N-1} P_i + B_{00} \right) = 0 \quad (7)$$

The loading of the dependent generator called slack generator (i.e.  $N$ th) can then be found by solving (7).

#### Economic emission dispatch

The economic emission dispatch problem for  $\text{NO}_x$  gases emission can be defined as

$$F_2 = \left( \sum_{i=1}^N F X_i(P_i) \right) = \left( \sum_{i=1}^N 10^{-2} (\alpha_i + \beta_i P_i + \gamma_i P_i^2) + \xi_i \exp(\lambda_i P_i) \right) \text{ Ton/h.} \quad (8)$$

where  $F_2$  is total amount of  $\text{NO}_x$  released from the system in (kg/h or ton/h);  $F X_i(P_i)$  is the  $i$ th generator's emission function for  $P_i$  output;  $\alpha_i$ ,  $\beta_i$ ,  $\gamma_i$ ,  $\xi_i$  and  $\lambda_i$  are the emission coefficients of  $i$ th generator. The above equation is minimized subject to the following constraints mentioned in (2) and (4).

#### Economic emission load dispatch

The EELD seeks a balance between cost and emission. The EELD problem can be formulated as,

$$\text{Minimize } C(f_1, f_n) \quad (9)$$

where 'n' can be 2 or 3 or more depending on number of objective function. This equation is minimized subject to the constraint as given in (2) and (4).

The economic load dispatch and emission dispatch problem are contradictory in nature as the economic load dispatch reduces the total fuel cost of the system, without any concern about the rate of emission. Economic emission dispatch, on the contrary, reduces the total emission from the system, which generally causes an increase in the system operating cost. As economic emission load dispatch (EELD) seeks a balance between the fuel cost and emission hazards simultaneously, therefore this problem may be considered as a multi-objective optimization problem.

The above mentioned multi-objective optimization can be solved using Fuzzy set theory along with any conventional optimization techniques [28,32], weighted sum method and many other techniques. Again, the above mentioned multi-objective problem can be solved after converting EELD problem to a single objective optimization problem by introducing the concept of price penalty factors (PPF) [39]. As per the concept of price penalty factor, the total operating cost of the system is the cost of generation plus the implied cost of emission. If number of objective function is two, i.e. when fuel cost and  $\text{NO}_x$  emission is considered, the overall objective function may be formulated with the help of PPF and represented as:

$$\text{Minimize } C = \sum_{i=1}^n \left[ w F_i(P_i) + (1-w) h E_i(P_i) \right] \quad (10)$$

Here 'h' is the price penalty factor which is blending the emission costs with the normal fuel costs and 'w' is the trade-off parameter in the range of [0,1]. This equation is minimized subject to demand constraint and generating capacity limits as given in (2) and (4). When the value of w is 1 the objective function represents fuel cost of generation function and when w is equal to 0, the objective function represents emission function only. It is very difficult to make a solution that will give the best compromising solution (BCS) which lie nearer to both of the best solution. The fuel cost increases and the emission cost decreases when w is reduced in steps from 1 to 0. The problem becomes purely EED that minimizes only the emissions when w is equal to 0.

The constrained optimization problem of Eq. (10) along with the constraints of (2) and (4) can be solved for optimal generations for a chosen value of w. The Pareto front based on the non-dominated solution can be obtained by solving the problem

several times with different  $w$  values. However it may not yield the best compromising solution, which may be defined as the one with equal percent deviations from the optimal solutions corresponding to ELD and EED. The BCS can be obtained simply by setting  $w$  as 0.5 [31], if the chosen  $h$  parameter does make fuel cost and emission cost components to the same level in the objective function. The optimization process attempts to give more importance to fuel cost than emission cost and vice versa, if the fuel cost component of Eq. (10) is larger than the equivalent emission cost. Besides, the fuzzy based strategies [28] and the methods based on competition [10] may not provide satisfactory results.

Recently, to find the best compromising solution a method is proposed by Rajasomashekar et al. [31]. The drawbacks of the existing approaches are overcome, after expressing bi-objective function of Eq. (10) in a modified way after normalizing the fuel cost and emission components with a view to endow with relatively equal significance to both the objectives. The modified overall objective function may be represented as:

$$\text{Min } C = w \left[ \frac{\sum_{i=1}^n F_i(P_i) - F_{1\min}}{F_{1\max} - F_{1\min}} \right] + (1 - w) \left[ \frac{\sum_{i=1}^n F_{Xi}(P_i) - F_{2\min}}{F_{2\max} - F_{2\min}} \right] \quad (11)$$

where  $F_{Xi}(P_i)$  is the value of emission and  $F_i(P_i)$  represents the total cost of generations. The values of  $F_{1\max}$ ,  $F_{1\min}$ ,  $F_{2\max}$ ,  $F_{2\min}$  can however be obtained after solving ELD and EED problems individually using (1) and (8) respectively, subject to the constraints of (2) and (4). As cost and emission functions are contradictory in nature. Therefore, solution of ELD problem will provide the value of  $F_{1\min}$ ,  $F_{2\max}$ . Similarly solution of EED problem will give the value of  $F_{1\max}$ ,  $F_{2\min}$ . The modified normalized representation of objective function for EELD problem has the following advantages [31]:

- (i) Eq. (11) eliminates the use of price penalty factor,  $h$  which is one of the advantages (as calculation procedure of PPF normally needs some approximation).
- (ii) Moreover, this new problem formulation offers best compromising solution (BCS) when  $w$  is set to 0.5 [31] and the overall solution process involves only three runs for solution of ELD, EED and EELD problems. But, fuzzy based strategies necessitate several solution runs with different  $w$  values. The existing approaches provide a solution, whose fuel cost is very close to the best fuel cost while keeping the emission components far away from the best emission point and vice-versa. This indicates that the relative importance given to both objectives are unequal. But according to [31], the new problem formulation (11) based optimization process gives almost equal importance to both the fuel cost and emission components and brings their values to lie in the same range. The amount by which best compromising solutions swerve from the global best fuel cost and emissions are calculated using the following indices:

(Fuel Cost Performance Index)FCPI

$$= \left[ \frac{\sum_{i=1}^n F_i(P_i) - F_{1\min}}{F_{1\max} - F_{1\min}} \right] \times 100$$

(Emission Cost Performance Index)ECPI

$$= \left[ \frac{\sum_{i=1}^n F_{Xi}(P_i) - F_{2\min}}{F_{2\max} - F_{2\min}} \right] \times 100$$

However, the relative significance between fuel cost and emissions can be varied by changing  $w$  in between 0 and 1 in the objective function of (11). It permits the system operator to decide on different preferences for the objectives according to system operating conditions.

In the present paper, (11) is used as the objective function and it is used for optimization subject to the constraints of (2) and (4), for finding best compromising solutions.

### Backtracking search optimization (BSA)

This section presents a modern optimization algorithm called backtracking search optimization (BSA) which has been recently proposed in [38].

The success of an optimization algorithm significantly depends on its exploration and exploitation abilities. BSA has both global exploration and local exploitation abilities. Global exploration ability means that the optimization algorithm effectively uses the entire search space, while local exploitation ability means that the optimization algorithm searches for the best solution near a new solution it has already discovered. BSA uses mutation operation through their exploration ability to acquire the new solutions that are needed to avoid local minimums in their first iterations. BSA uses a non-uniform crossover strategy that is more complex than the crossover strategies used in other algorithms to maintain the diversity of the trial population.

In the following subsections, major components of the BSA based design, i.e., initialization, selection-I, mutation, crossover and selection-II are described. Then sequential steps of BSA to solve ELD problem are also presented in the subsequent subsections.

#### Initialization of BSA

In initialization step of BSA there is random generation of population ( $PoP$ ) within their marginal boundary i.e.  $PoP_{\max}$  and  $PoP_{\min}$ .

```

for i = 1: PopSize
    for j = 1: D
        PoP(i,j) = PoP_max(j) - rand * (PoP_max(j) - PoP_min(j))
    end
end
    
```

$PoP$  is the target individuals,  $PopSize$  and  $D$  are the maximum population size and dimension of the population set.

#### Selection I

After initialization step, *selection I* is a very important step to generate historical population set ( $oldPoP$ ). The initial historical population is determined as the same procedure of (12) follows:

```

for i = 1: PopSize
    for j = 1: D
        oldPoP(i,j) = PoP_min(j) + rand * (PoP_max(j) - PoP_min(j))
    end
end
    
```

From the starting of each iteration, each elements of historical population set is updated using (14).

```

if rand < rand
    oldPoP(i,j) = PoP(i,j)
end
    
```

where  $rand$  is taken any random number (0–1). Randomly selected historical population set are stored as a memory in BSA algorithm until it is not changed by getting better fitness value. After  $oldPoP$

is formed, following equation is used to change randomly shuffling the order of the individuals in *oldPoP*:

$$\text{oldPoP} = \text{oldPoP}(\text{randperm}(\text{PopSize}),:); \quad (15)$$

#### Mutation

Mutation is the important process in BSA, through which the trial population matrix, *mutant* is generated by using (16).

$$\text{Mutant} = \text{PoP} + \text{FC} * (\text{oldPoP} - \text{PoP}) \quad (16)$$

*FC* controls the amplitude of the search-direction matrix (*oldPoP* – *PoP*) and its value is taken ( $3 * \text{rdn}$ ). Due to the involvement of historical population the trial population set; *mutant* is taking some advantage of generation of its experiences from previous generations.

#### Crossover

Trial population set (*T*) form in mutation as *mutant* is taken as the initial value in crossover section. Best fitness value in trial individuals is taken as target population individuals. Crossover section has two parts. Firstly generate a set of binary integer-valued matrix (*Bmap*) arbitrary whose size is same as the population size, i.e.  $\text{PopSize} \times D$ . *Bmap* is the indicator which indicate that the trial population *T* is either updated with population set (*PoP*) or not. i.e. if  $Bmap(i,j) = 1$ , then  $T(i,j) := PoP(i,j)$ . Where  $:=$  is an update operator. A mix rate parameter (*mixrate*) in BSA's crossover process is used to controls the number of elements of individuals that will mutate in a trial by using ( $\text{mixrate} * \text{rand} * D$ ). The function of the *mixrate* is quite different from the crossover rate used in different algorithm. Corresponding algorithm for Selection-I, Mutation and Crossover steps together is given below:

---

```

Define mixrate
oldPoP = oldPoP(randperm(PopSize),:);
map = zeros(PopSize,D);
if rand < rand,
    for i = 1: PopSize,
        u = randperm(D);
        map(i,u(1:ceil(mixrate * rand * D))) = 0;
    end
else
    for i = 1: PopSize,
        map(i,randi(D)) = 0;
    end
end
Mutant = pop + (map.* FC).*( historical_pop-pop);

```

---

Each set of trial population (*T*) must go through the boundary control mechanism if any value violet their operating limit.

```

for i = 1:PopSize
    for j = 1:D
        k = rand < rand;
        if PoP(i,j) < PoP_min(j)
            if k
                PoP(i,j) = PoP_min(j);
            else
                PoP(i,j) = rand * (PoP_max(j) - PoP_min(j)) + PoP_min(j);
        end
    end
end

```

```

if PoP(i,j) > PoP_max(j),
    if k
        PoP(i,j) = PoP_max(j);
    else
        PoP(i,j) = rand * (PoP_max(j) - PoP_min(j)) + PoP_min(j);
    end
end
T(i,:) = PoP(i,:);
end

```

---

#### Selection II

BSA's *Selection-II* is a stage where all the data are collected and the fitness values of trial population (*T*) and population set (*PoP*) are compared and an updated corresponding population set comes into existence. If the best individual of Population (*Pbest*) has a better fitness value than the global minimum value obtained so far by BSA, the global minimizer is updated to be *Pbest*, and the global minimum value is updated to be the fitness value of *Pbest*. The structure of BSA is quite simple; thus, it is easily adapted to different numerical optimization problems. The pseudo codes for all above-mentioned steps are available in [38].

#### BSA algorithm for economic emission load dispatch problem

In this subsection, the procedure to implement the BSA algorithm for solving the EELD problems has been described. Description of the flow chart of BSA algorithm is shown in Fig. 1. This algorithm is also used to handle with the equality and inequality constraints of the EELD problems.

1. *Representation of the population set X*: Since the appraisal variables for EELD problem are real power output of the generators with consideration of cost minimization and emission minimization, they are used to represent the individual population set. Each individual element of the population set represents the real power output of each generator. For initialization, choose the number of generator unit *m* and the total number of population set, *PopSize*. The complete population set is represented in the form of the following matrix:

$$X = X_i = [X_1, X_2, X_3, \dots, X_{\text{PopSize}}] \text{ where } i = 1, 2, \dots, \text{PopSize}$$

In case of EELD problem, each population set is presented as:

$$X_i = [X_{i1}, X_{i2}, \dots, X_{im}] = [Pg_{ij}] = [Pg_{i1}, Pg_{i2}, \dots, Pg_{im}];$$

where  $j = 1, 2, \dots, m$ . Each population set is one of the possible solutions for the EELD problem. The element  $X_{ij}$  of  $X_i$  is the *j*th position component of population set *i*.

2. *Initialization of the population set*: Each individual element of the population structure matrix is initialized randomly within their effective real power upper and lower limit of power generations based on (4).
3. *Evaluation of objective function*: In case of EELD problems, objective function of each population set is represented by the total fuel cost of generation and emission for all the generators of that given population set. The ELD problem is calculated using (1) for the system having valve point loading. In case of EED problems it is calculated using (8) for the system having complex emission characteristic. Using (11) objective function is

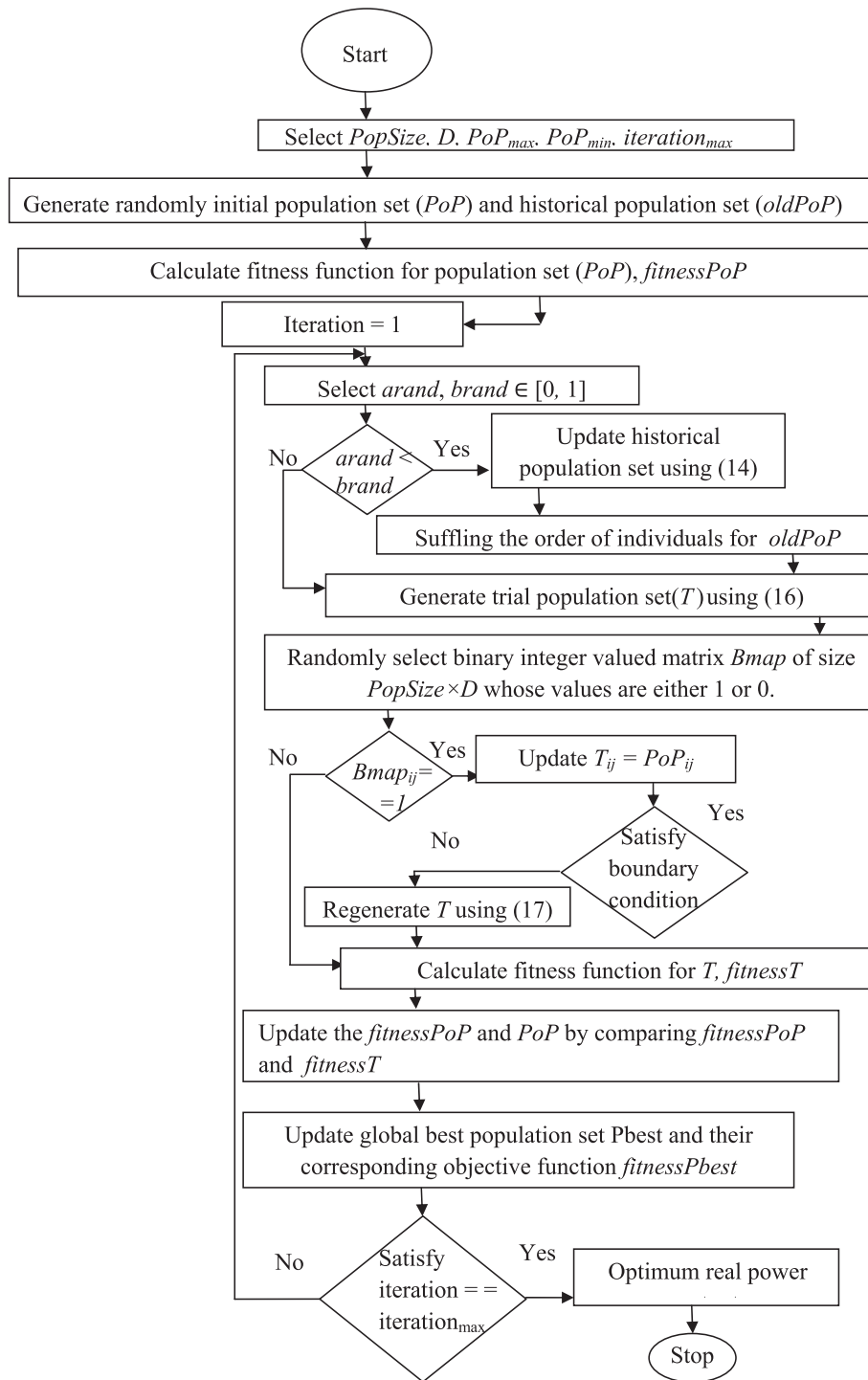


Fig. 1. Flow chart of proposed BSA technique.

calculated for different values of  $w$ , in case of EELD problems. As (11) contains  $F_{1max}, F_{1min}, F_{2max}, F_{2min}$  terms, therefore solution of ELD and EED problem is required to get the values of those terms. Therefore, it is required to run ELD and EED programs first before running EELD program.

Now the steps of algorithm to solve EELD problems are given below.

Step (1) For initialization, choose No. of generator units,  $m$ ; number of population set,  $PopSize$ . Set no. of elite molecule sets,

' $p$ '. Specify maximum and minimum capacity of each generator, power demand, B-coefficients matrix for calculation of transmission loss. Also initialize the BSA parameter  $mixrate$ .  $PoP_{max}$  and  $PoP_{min}$  are taken as maximum and minimum capacity of each generator. Set maximum number of iterations,  $Iter_{max}$ .

Step (2) Initialize the value of  $w$ . Set its starting value as  $w = 0$ . Step (3) Initialize each element of a given population set of  $X$  matrix using (12) and the concept mentioned in "Initialization of the population set". Each population set of  $X$  matrix should satisfy equality constraint of (2) using the concept of

slack generator as mentioned in Section 'Economic load dispatch'.

Step (4) Calculate the objective function for each population set of the  $X$  matrix based on (11).

Step (5) Based on the objective function values identify the best population set, which give best value of (11) for the specified value of  $w$ . Keep top ' $p$ ' population sets unchanged after individual iteration, without making any modification on it.

Step (6) Randomly initialize the historical population set ( $oldPoP$ ) using (13) in the same procedure mentioned in step (3) with all the satisfying respective constraints. Create two random number  $arand, brand \in [0, 1]$ . Each element of historical population set is updated using (14) for each iterations if  $arand < brand$  condition satisfy, otherwise keep the old historical population set. After forming new set of historical population set, the order of each individual, i.e. each set of active power generation are shuffled randomly using (15).

Step (7) Initial value of population matrix  $mutant$  in mutation step can be generated using (16).

Step (8) Initial value in crossover process is  $mutant$  and is considered as trial population set ( $T$ ). Create a randomly selected binary integer valued matrix  $Bmap$  of size  $PopSize \times m$  whose values are either 1 or 0. Update the values of  $T$  to the pertinent individuals of  $X$ . The pseudo code for crossover operation is given in Section 'Crossover'. Each set of trial population ( $T$ ) must satisfy the concept of slack generator as mentioned in Section 'Economic load dispatch'. If any set violate their operating limit of slack generator then generate a new set using

(17). Calculate the objective function for each set of trial population.

Step (9) compare the values of the objective function for population set and trial population set. If the trial population set has better fitness value then update the population set ( $X$ ). If the best individual of Population ( $Pbest$ ) has a better fitness value than the global minimum value obtained so far by BSA, the global minimizer is updated to be  $Pbest$ , and the global minimum value is updated to be the fitness value of  $Pbest$ .

Step (10) Terminate the iterative process, if current iteration is greater than or equal to the maximum iteration ( $Iter_{max}$ ). Store the best power outputs obtained in an array " $Optimal Set$ "; otherwise repeat the steps 4 to 9.

Step (11) Increment the value of ' $w$ ' in step of 0.05 and repeat the steps starting from step 3 to step 10, until the value of ' $w$ ' reaches to 1.

Step (12) *Best Compromising Solution*: Calculate the value of fuel cost of generation and emission for each solution sets, those are obtained for different values  $w$  and stored in the array " $Optimal Set$ ". Use (1) and (8) to calculate fuel cost of generation and emission respectively for each set. Calculate FCPI and ECPI using the equations mentioned at the end of Section 'Economic emission load dispatch', for each fuel cost of generation and emission set. Evaluate the absolute value of difference between FCPI and ECPI for each fuel cost of generation and emission set. The set that attains minimum absolute value of difference between FCPI and ECPI is chosen as the best compromising solution. The fuel cost of generation and emission values

**Table 1**  
Minimum fuel cost and minimum emission obtained by BSA for Test system-1 (PD = 1200 MW).

Units	Power outputs (MW)					
	Minimum cost			Minimum emission		
	BSA	QOTLBO [40]	DE [40]	BSA	QOTLBO [40]	DE [40]
1	79.6762	79.5547	84.4354	125.0000	125.0000	125.0000
2	88.7507	88.8977	93.3638	150.0000	150.0000	150.0000
3	210.0000	210.0000	225.0000	201.2684	201.2679	201.1816
4	225.0000	224.9944	209.9995	199.3690	199.3701	199.5454
5	325.0000	324.9708	325.0000	287.9713	287.9708	287.6191
6	324.9927	324.9977	314.9998	286.5499	286.5498	286.8137
Total generation (MW)	1253.42	NA*	NA	1250.1586	NA	NA
Loss (MW)	53.42	53.42	NA	50.1586	50.1586	NA
Cost (\$/h)	63,976	63,977	64,083	65,992	65,992	65,991
Emission (lb/h)	1360.1	1360.1	1345.6	<b>1240.6</b>	1240.6	1240.7

\* NA: Data not available.

**Table 2**  
Comparison of the best compromising solutions for Test system-1 (PD = 1200 MW).

Units	Power outputs (MW)					
	SPEA-2 [40]	NSGA-II [40]	PDE [40]	MODE [40]	QOTLBO [40]	BSA
P1	104.1573	113.1259	107.3965	108.6284	107.6485	104.0108819101
P2	122.9807	116.4488	122.1418	115.9456	121.4066	117.4295760078
P3	214.9553	217.4191	206.7536	206.7969	206.1323	207.5810795585
P4	203.1387	207.9492	203.7047	210.0000	205.5255	206.6238337780
P5	316.0302	304.6641	308.1045	301.8884	306.6997	309.0648331284
P6	289.9396	291.5969	303.3797	308.4127	304.0893	306.9982692688
Cost (\$/h)	64,884	64,962	64,920	64,843	64,915	<b>64766.8227149105</b>
Emission (lb/h)	1285	1281	1281	1286	1281	<b>1289.5856051925</b>
FCPI	NA	NA	NA	NA	46.5509	<b>39.2273</b>
ECPI	NA	NA	NA	NA	33.8075	<b>40.9921</b>
Difference	NA	NA	NA	NA	12.7433	<b>1.7648</b>

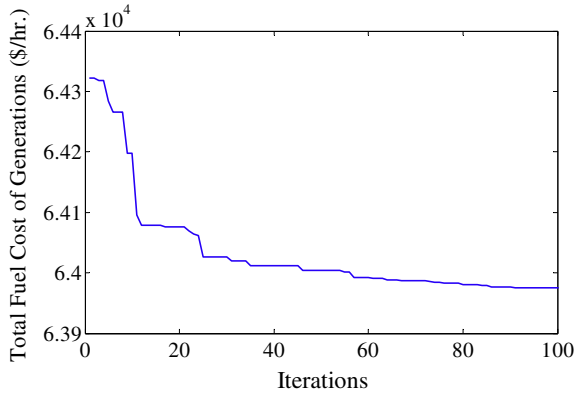


Fig. 2. Convergence characteristic for fuel cost minimization (Test system-1, PD = 1200 MW), obtained by BSA.

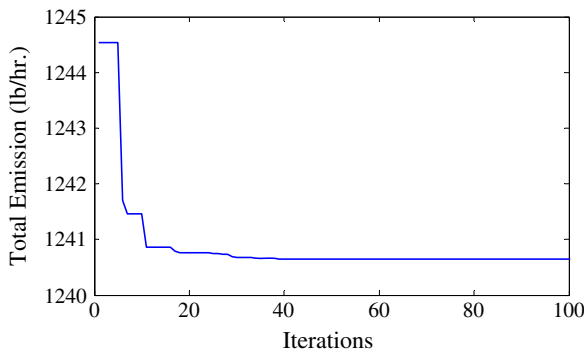


Fig. 3. Convergence characteristic for emission minimization (Test system-1, PD = 1200 MW), obtained by BSA.

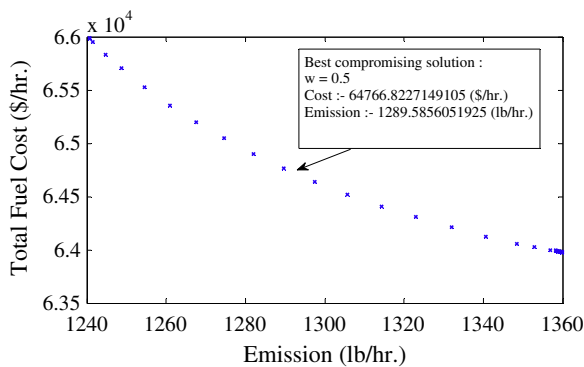


Fig. 4. Trade-off curve obtained by BSA for Test system-1.

associated with that set represent the best compromising fuel cost and emission. Find the power output of that set from the “Optimal Set” array.

Interested readers may refer [38], which contains the detail steps of the BSA Algorithm.

**Numerical examples and simulation results**

The BSA algorithm has been applied to three different test systems with varying degree of complexity for verifying its feasibility. Transmission loss has been calculated using loss coefficient matrix. The program has been written in MATLAB-7.5 language and executed on a 2.5 GHz Intel Dual Core personal computer with 2-GB RAM.

*Description of the test systems*

- (1) Test system 1: A small test system with 6 generating unit having fuel cost function is considered. The input data like: fuel cost coefficients, emission coefficients, operating limits of generators have been adopted from [40]. The transmission loss coefficients are taken from [41]. The load demand is 1200 MW. The minimum fuel cost, minimum emission results obtained by proposed BSA has been presented in Table 1. Minimum fuel cost, minimum emission obtained by BSA are 63976 \$ and 1240.6 lb respectively. Comparisons of best compromising results obtained by BSA, SPEA-2 [40], NSGA-II [40], PDE [40], MODE [40] and QOTLBO [40] have been shown in Table 2. For above mentioned methods, calculated values of FCPI and ECPI with respect to minimum cost and emission results of BSA and other method are also presented in Table 2. The lower difference of 1.7648 between FCPI and ECPI for the test system guarantees the soundness of the BSA in offering best com-

**Table 4**  
Minimum fuel cost and minimum emission obtained by BSA for Test system-2 (PD = 2000 MW).

Units	Power outputs (MW)	
	Minimum cost	Minimum emission
1	55.0000000	55.0000000
2	80.0000000	80.0000000
3	106.9395807	81.1341690
4	100.5762919	81.3637414
5	81.5019997	160.0000000
6	83.0209509	240.0000000
7	300.0000000	294.4850788
8	340.0000000	297.2701080
9	470.0000000	396.7657221
10	470.0000000	395.5763320
Total generation (MW)	2000.000000	2000.000000
Loss (MW)	87.0388230877603	81.595151187074080
Cost (\$/h)	<b>111497.6308105137</b>	116412.4441154830
Emission (lb/h)	4572.1939661792	<b>3932.2432691519</b>

**Table 3**  
Minimum, average, maximum best compromise solution obtained by BSA over 50 trials (Test system-1, PD = 1200 MW).

Method	Total cost (\$/h)			Total emission (lb/h)			Average simulation time (s)	No. of hits to optimum solution	Standard deviation
	Max.	Min.	Average	Max.	Min.	Average			
BSA	64766.8227150	64766.8227150	64766.8227150	1289.5856052	1289.5856052	1289.5856052	0.61	50	0.0000

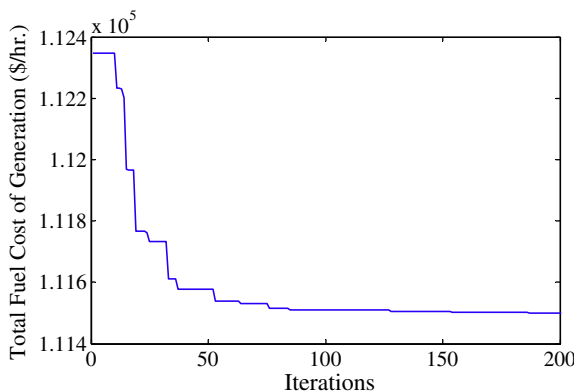


**Table 5**  
Comparison of the best compromising solutions for Test system-2 (PD = 2000 MW).

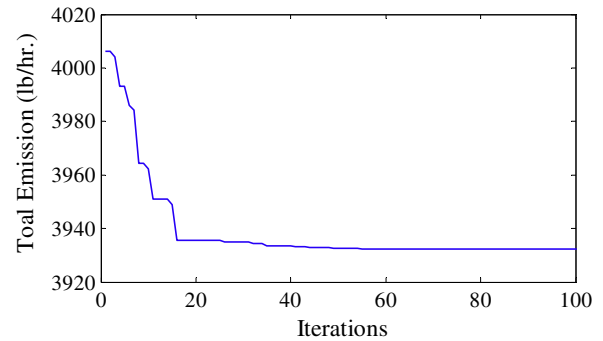
Units	Power outputs (MW)					
	MODE [42]	PDE [42]	NSGA-II [42]	SPEA 2 [42]	GSA [42]	BSA
P1	54.9487	54.9853	51.9515	52.9761	54.9992	55.0000000000
P2	74.5821	79.3803	67.2584	72.8130	79.9586	80.0000000000
P3	79.4294	83.9842	73.6879	78.1128	79.4341	85.6466447236
P4	80.6875	86.5942	91.3554	83.6088	85.0000	84.1269519822
P5	136.8551	144.4386	134.0522	137.2432	142.1063	136.4904620307
P6	172.6393	165.7756	174.9504	172.9188	166.5670	155.5642036545
P7	283.8233	283.2122	289.4350	287.2023	292.8749	299.9999999980
P8	316.3407	312.7709	314.0556	326.4023	313.2387	316.6806634004
P9	448.5923	440.1135	455.6978	448.8814	441.1775	434.1352371470
P10	436.4287	432.6783	431.8054	423.9025	428.6306	436.5834194155
Cost (\$/h)	$11,348 \times 10^5$	$1.1351 \times 10^5$	$1.1354 \times 10^5$	$1.1352 \times 10^5$	$1.1349 \times 10^5$	113126.7514673425
Emission (lb/h)	4124.9	4111.4	4130.2	4109.1	4111.4	4146.7285586228
FCPI	40.33	40.94	41.56	41.15	40.54	<b>33.1471524889384</b>
ECPI	30.12	28.01	30.94	27.65	28.01	<b>33.5158422104408</b>
Difference	10.21	12.93	10.62	13.50	12.53	0.3686897215024

promising solution. Convergence characteristics of the 6 generators system for minimum fuel cost, minimum emission in case of BSA are shown in Figs. 2 and 3 respectively. Trade-off curve obtained by BSA for different values of  $w$  using objective function of (11) is shown in Fig. 4. As BSA is a stochastic simulation method, randomness in the simulation result is understandable. To find out the optimum results many trials therefore are required. Minimum, average and maximum compromise solution obtained by BSA over 50 trials are presented in Table 3. Again EELD is a real time problem, so it is desirable that each run of the program should reach close to optimum solution. Table 3 clearly indicate outstanding success rate, 100% of the BSA. From Table 3, it is clear that the average cost and emission for the compromising solutions (63976 \$, 1240.6 lb) achieved by BSA is same as its minimum result (63976 \$, 1240.6 lb). Moreover, average simulation time of BSA is 0.61 s. which is the proof of quite attractive computational efficiency of BSA. All these results denote the robustness and superiority of BSA.

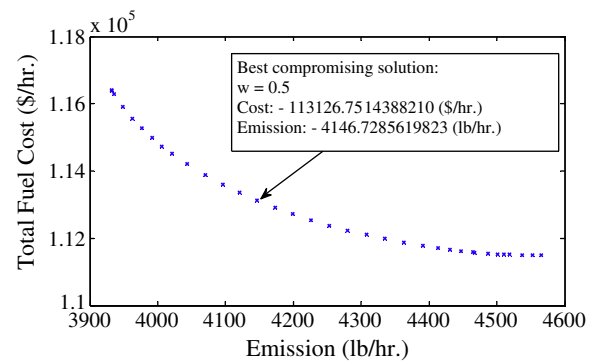
- (2) Test system 2: A 10 generator system having the effects of valve-point loading on quadratic fuel cost function and emission level functions are considered. The input data like: cost coefficients, emission coefficients, operating limits of generators and loss coefficients have been adopted from [42]. The load demand is 2000 MW. The minimum fuel cost, minimum NO<sub>x</sub> emission results obtained by proposed BSA



**Fig. 5.** Convergence characteristic for fuel cost minimization (Test system-2, PD = 2000 MW), obtained by BSA.



**Fig. 6.** Convergence characteristic for emission minimization (Test system-2, PD = 2000 MW), obtained by BSA.



**Fig. 7.** Trade-off curve obtained by BSA for Test system-2.

has been presented in Table 4. Minimum fuel cost, minimum NO<sub>x</sub> emission obtained by BSA are 111497.6308105137 \$ and 3932.2432691519 lb respectively. Comparisons of best compromising results obtained by BSA, MODE [42], PDE [42], NSGA-II [42], SPEA [42], GSA [42] and BSA have been shown in Table 5. For above mentioned methods, calculated values of FCPI and ECPI with respect to minimum cost and emission results of BSA are also presented in Table 5. The lower difference of 0.3686897215024 between FCPI and

**Table 6**  
Minimum, average, maximum best compromise solution obtained by BSA over 50 trials (Test system-2, PD = 2000 MW).

Method	Total cost (\$/h)		Total emission (lb/h)		Average simulation time (s)	No. of hits to optimum solution	Standard deviation
	Max.	Min.	Max.	Min.			
BSA	113126.7514673425	113126.7514673425	4146.7285586228	4146.7285586228	<b>0.3182</b>	<b>50</b>	<b>0.0000</b>

ECPI for the test system ensures the validity of the BSA in offering best compromising solution. Convergence characteristics of the 10 generators system for minimum fuel cost, minimum NO<sub>x</sub> emission in case of BSA are shown in Figs. 5 and 6 respectively. Trade-off curve obtained by BSA for different values of  $w$  using objective function of (11) is shown in Fig. 7. Minimum, average and maximum compromise solution obtained by BSA over 50 trials are presented in Table 6. Again EELD is a real time problem, so it is desirable that each run of the program should reach close to optimum solution. Table 6 clearly point to inclusive success rate, 100% of the BSA. From Table 6, it is clear that the average cost and emission for the compromising solutions achieved by BSA (111497.6308105137 \$, 3932.2432691519 lb) is same as its minimum result (111497.6308105137 \$, 3932.2432691519 lb). Moreover, average simulation time of BSA is 0.3182 s. which is the proof of quite attractive computational efficiency, robustness and superiority of BSA.

- (3) Test system 3: A system with 40 generator units with valve-point loading and NO<sub>x</sub> emission has been considered in this case. The unit cost and emission coefficients, operating limits are as in [28]. Transmission loss has not been considered here. The simulation results of minimum fuel cost, minimum NO<sub>x</sub> emission and best compromising solutions, FCPI and ECPI obtained by BSA, MBFA [28] for a demand of 10500 MW have been presented in Table 7. Minimum fuel cost and minimum emission obtained by BSA is slightly better than those obtained by MBFA. Moreover, difference between FCPI and ECPI, obtained by BSA and other methods are shown in Table 8. The difference value between FCPI and ECPI for the test system guarantees the superiority of BSA with respect to MBFA and other methods in offering best compromising solutions. Convergence characteristic obtained by BSA for minimum fuel cost and minimum NO<sub>x</sub> emission are shown in Figs. 8 and 9 respectively. The trade-off curve for the test system obtained by BSA is shown in Fig. 10. The figure shows that the best compromising solution of 124187.8724140377 \$/h and 233544.8777308412 Ton/h is obtained by BSA when  $w = 0.5$ . Minimum, average and maximum best compromising solutions obtained by BSA, over 50 trials are presented in Table 8. Same minimum, average and maximum best compromising solution (124187.8724140377 \$/h and 233544.8777308412 Ton/h) have been obtained by BSA over 50 trials in Table 9. Results reflect that the BSA is very robust tool for solving complex ELD, EED and EELD problems as its convergence rate is 100%. The simulation time required by BSA, to reach minimum solution is 0.49 s, which is better than the result (63.21 s) presented in [28]. The simulation study clearly indicates that the BSA is able to offer superior performance than MBFA and many other techniques.

Hence, it may be concluded that the BSA is a computationally efficient, fast and robust optimization technique to solve complex small as well as large EELD problems (see Table 9).

#### Determination of population size for BSA

Change in population size also affects the performance of the BSA. Large or small value of population size may not give the optimum value. For each population size of 20, 50, 100, 150 and 200, 50 trials have been run using test system-3. Table 10 shows the performance of the BSA for different population sizes. A population size of 50 resulted in achieving global solutions more consistently and efficiently for the test system.

**Table 7**  
Minimum fuel cost, minimum emission and best compromising solution for Test system-3 (PD = 10500 MW).

Units	Power outputs for minimum cost (MW)		Power outputs for minimum emission (MW)		Power outputs for best compromising solution (MW)	
	MBFA [28]	BSA	MBFA [28]	BSA	MBFA [28]	BSA
1	114.0000	110.799825	114.0000	114.000000	–	111.0281074489
2	110.8035	110.799825	114.0000	114.000000	–	110.9842527761
3	97.4002	97.399913	120.0000	120.000000	–	97.5185163315
4	179.7333	179.733100	169.3671	169.368008	–	179.6234867443
5	87.8072	87.799905	97.0000	97.000000	–	87.9013924217
6	140.0000	140.000000	124.2630	124.257413	–	139.9361776473
7	259.6004	259.599650	299.6931	299.711391	–	299.9973224543
8	284.6002	284.599650	297.9093	297.914857	–	284.8178332017
9	284.6006	284.599650	297.2578	297.260104	–	284.6970405234
10	130.0000	130.000000	130.0007	130.000000	–	130.0010374829
11	168.7999	94.000000	298.4210	298.410143	–	243.5997080644
12	168.7998	94.000000	298.0264	298.026013	–	243.5860985318
13	214.7598	214.759790	433.5590	433.557638	–	394.3575792255
14	304.5195	394.279370	421.7360	421.728405	–	394.0842868786
15	394.2794	394.279370	422.7884	422.779645	–	394.2627488904
16	394.2794	394.279370	422.7841	422.779650	–	394.3353474691
17	489.2794	489.279370	439.4078	439.412858	–	489.2357501688
18	489.2794	489.279370	439.4132	439.402890	–	489.2747001725
19	511.2795	511.279370	439.4111	439.412857	–	511.2517439767
20	511.2795	511.279370	439.4155	439.412855	–	421.4733107913
21	523.2794	523.279370	439.4421	439.446400	–	433.4451792880
22	523.2794	523.279370	439.4587	439.446402	–	433.5130774375
23	523.2796	523.279370	439.7822	439.772065	–	433.5401290243
24	523.2794	523.279370	439.7697	439.772065	–	521.7718624912
25	523.2795	523.279370	440.1191	440.111766	–	433.6479218298
26	523.2796	523.279370	440.1219	440.111765	–	433.6179333566
27	10.0001	10.000000	28.9738	28.993703	–	10.0987179581
28	10.0002	10.000000	29.0007	28.993700	–	10.0803506860
29	10.0002	10.000000	28.9828	28.993702	–	10.0058397903
30	89.5070	87.799905	97.0000	97.000000	–	88.0450861104
31	190.0000	190.000000	172.3348	172.331904	–	189.9999995943
32	190.0000	190.000000	172.3327	172.331906	–	189.9848931963
33	190.0000	190.000000	172.3262	172.331903	–	189.9779636598
34	164.8026	164.799825	200.0000	200.000000	–	199.9723618480
35	164.8035	194.397778	200.0000	200.000000	–	199.9821976549
36	164.8292	200.000000	200.0000	200.000000	–	199.9999110120
37	110.0000	110.000000	100.8441	100.838378	–	89.2084771722
38	110.0000	110.000000	100.8346	100.838376	–	109.9997538737
39	110.0000	110.000000	100.8362	100.838380	–	109.9719371585
40	511.2795	511.279370	439.3868	439.412855	–	511.1699656569
Total generation (MW)	10500.00	10500.00	10500.00	10500.00	–	10500.00
Fuel Cost (\$/h)	121415.653	<b>121412.535523029</b>	129995.000	129995.2711863580	123638.0000	<b>124187.8724140377</b>
Emission (Ton/h)	356424.497	359901.3816251208	176682.269	<b>176682.2646796508</b>	188963.0000	<b>233544.8777308412</b>
FCPI	0	0	100	100	25.9034	<b>32.3362736587361</b>
ECPI	100	100	0	0	6.8324	<b>31.0353057034512</b>
Difference	100	100	100	100	19.0710	<b>1.3009679552849</b>

**Table 8**  
Best compromising solution for Test system-3 (PD = 10500 MW).

Methods	BSA	MBFA [28]	MODE [42]	PDE [42]	NSGA-II [42]	SPEA [42]	GSA [42]
Fuel cost (\$/h)	124187.8724	123638.0000	125,790	125,730	125,830	125,810	125,780
Emission (Ton/h)	233544.8777	188963.0000	211,190	211,770	210,950	211,100	210,930
FCPI	32.3362	25.9034	51.0031	50.3040	51.4692	51.2361	50.8866
ECPI	31.0353	6.8324	18.8341	19.1507	18.7031	18.7850	18.6922
Difference	<b>01.3009</b>	19.0710	32.1690	31.1533	32.7660	32.4511	32.1943

#### Determination of parameters for BSA

Tuning of different parameter like, *mixrate* and *FC* is required to search out optimum solution using BSA algorithm. For

different values of these parameters, difference between FCPI and ECPI (based on best compromising solutions) are evaluated for 40 generators system (Test System-3) and are presented in Table 11.

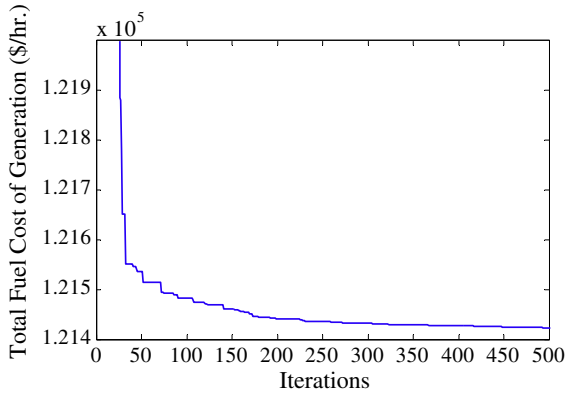


Fig. 8. Convergence characteristic for fuel cost minimization (Test system-3, PD = 10,500 MW), obtained by BSA.

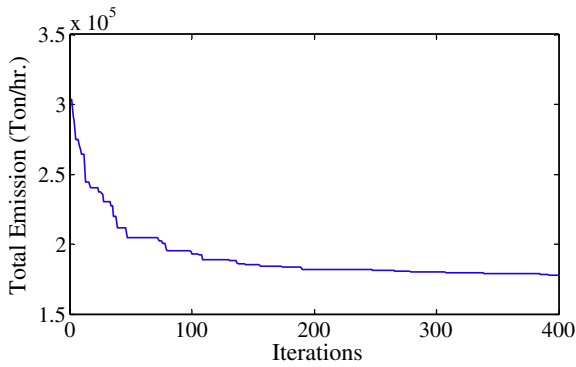


Fig. 9. Convergence characteristic for emission minimization (Test system-3, PD = 10500 MW), obtained by BSA.

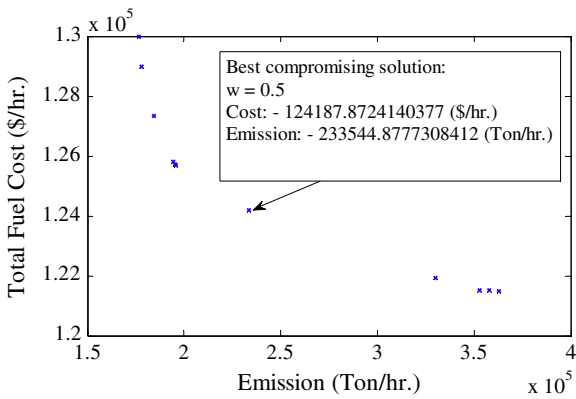


Fig. 10. Trade-off curve obtained by BSA for Test system-3.

**Conclusion**

In this paper, an enhanced BSA algorithm is proposed to solve both small and large EELD problems. More complex fuel cost characteristic is considered (such as valve point loading is considered). In order to validate the efficiency of BSA, three different test systems are taken to testify the proposed method. Simulation results obtained from the proposed approach have been compared with those from previous methods. The proposed approach obtained results that are better than those obtained by other algorithms considering single as well as multi-objective functions. The results

Table 9 Minimum, average, maximum best compromise solution obtained by BSA over 50 trials (Test system-3, PD = 10500 MW).

Method	Total cost (\$/h)		Total emission (Ton/h)		Average simulation time (s)	No. of hits to optimum solution	Standard deviation
	Max.	Min.	Max.	Min.			
BSA	124187.8724140377	124187.8724140377	233544.8777308412	233544.8777308412	0.49	50	0.0000

**Table 10**

Effect of population structure size on performance of BSA, based on difference between FCPI and ECPI for Test system-3.

Population structure size	No. of hits to best solution	Simulation time (s)	Max. difference	Min. difference	Average difference
20	48	2.08	1.301781	1.301507	1.301518
<b>50</b>	<b>50</b>	<b>2.59</b>	<b>1.300968</b>	<b>1.300968</b>	<b>1.300968</b>
100	45	2.94	1.301172	1.300984	1.301003
150	44	3.82	1.302897	1.302173	1.302260
200	41	5.21	1.325040	1.302579	1.30662198

**Table 11**

Effect of different parameters on performance of BSA, based on difference between FCPI and ECPI for Test system-3.

FC	Mixrate					
	1.0	0.9	0.7	0.5	0.3	0.1
1.9	1.302580	1.302878	1.303146	1.303741	1.304575	1.304811
1.6	1.301548	1.302007	1.302570	1.302780	1.302981	1.303267
1.3	1.300999	1.301081	1.301157	1.301249	1.301558	1.301658
1.0	<b>1.300968</b>	1.301007	1.301059	1.301137	1.301255	1.301477
0.8	1.301047	1.301149	1.301230	1.301583	1.301669	1.301989
0.6	1.301854	1.302358	1.302541	1.302630	1.302971	1.303252
0.4	1.302407	1.303501	1.303620	1.303819	1.304122	1.306540
0.3	1.305870	1.306511	1.308434	1.309076	1.309507	1.311214
0.2	1.309015	1.310789	1.311873	1.315766	1.316710	1.319877
0.1	1.315222	1.318288	1.320047	1.324100	1.325794	1.335740

show that the proposed approach can obtain a diversity preserving Pareto optimal solutions. Moreover, the non-dominated solutions in the obtained Pareto optimal set are well distributed and have good convergence characteristics. The results also allow the decision maker to consider this proposed approach to solve low emission EED problem. It has been also observed that the BSA has the ability to converge to quality solution within very short duration of time, in a computationally efficient manner and has better and stable convergence characteristics compared to other optimization techniques. Due to its promising performances, the BSA method seems to be a significant means to solve several other complex optimization problems in future.

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