Teaching Learning Based Optimization for Different Economic Dispatch Problems

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Abstract— This paper presents a teaching learning based algorithm (TLBO) to solve economic load dispatch (ELD) problems involving different linear, non-linear constraints. The problem formulation also considered the non-convex objective functions including the effect of valve-point loading, multi-fuel option of large-scale thermal plants.Many difficulties such as multimodality, dimensionality and differentiability are associated with the optimization of large scale non-linear constraints based non-convex economic load dispatch problems. TLBO is a population based technique which implements a group of solutions to proceed for the optimum solution. TLBO uses two different phases 'Teacher Phase' and 'Learner Phase'. TLBO uses the mean value of the population to update the solution. Unlike other optimization techniques TLBO does not require any parameters to be tuned, thus making the implementation of TLBO simpler. TLBO uses the best solution of the iteration to change the existing solution in the population thereby increasing the convergence rate. Therefore, in the present paper Teaching–Learning-Based Optimization (TLBO) is applied to solve such type of complicated problems efficiently and effectively in order to achieve superior quality solution in computationally efficient way. Simulation results show that the proposed approach outperforms several existing optimization techniques. Results also proved the robustness of the proposed methodology.

Keywords—Economic Load Dispatch, Prohibited operating zone, Ramp rate limits, Teaching-Learning Optimization, Valve-point loading

1. INTRODUCTION

Economic Load Dispatch is the process of allocating generation among the available generating units, considering the most efficient, reliable and low cost operation of a power system provided load demand and other operational constraints are satisfied. Its main aim is to minimize the total cost of generations while satisfying the operational constraints of the available thermal power generation resources. Initially, the traditional techniques [1] have been applied to solve ELD problems. Linear programming method [2] is fast and reliable but it has also some drawback. Classical optimization

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techniques are excellent for uni-modal and continuous functions. In case of these methods, the essential assumption is that, the incremental cost and emission curves of the generating units are monotonically increasing or piece-wise linear. A practical ELD problem sometimes considers effect of valve-point loading, Ramp-rate limits, Prohibited operating zones, multi-fuel options etc. into consideration. Due to all these practical effects the resulting ELD problems become totally non-convex optimization problem. Therefore, in some cases these methods converge to locally optimal solution and not in global. Dynamic Programming (DP) approach was proposed by Wood and Wollenberg, [3] to solve ELD problems. It imposes no restrictions on the characteristics of the generating units. It suffers from the curse of dimensionality and also increases execution time with the increase of system size.

Several attempts have been made to solve ELD problems using various soft computing techniques, such as genetic algorithm (GA) [4-5], particle swarm optimization (PSO) [6], An Colony Optimization (ACO) [7], evolutionary programming (EP) [8], simulated annealing (SA) [9], Differential Evolution (DE) [10], Artificial Immune System (AIS) [11], Bacterial Foraging Algorithm (BFA) [12], Biogeography-based Optimization (BBO) [13] etc. The above-mentioned techniques have been already proved very fast, reasonable nearly global optimal solution in solving nonlinear ELD problems without any restriction on the shape of the cost curves. Recently, different hybridization and modification of GA, EP, PSO, DE, BBO like improved GA with multiplier updating (IGA-MU) [14], hybrid genetic algorithm (GA)-pattern search (PS)-sequential quadratic programming (SQP) (GA-PS-SQP) [15], improved fast evolutionary programming (IFEP) [16], new PSO with local random search (NPSO_LRS) [17], adaptive PSO (APSO) [18], self-organizing hierarchical PSO (SOH-PSO) [19], improved coordinated aggregation based PSO (ICA-PSO) [20], improved PSO [21], combined particle swarm optimization with real-valued mutation (CBPSO-RVM)[22], DE with generator of chaos sequences and sequential quadratic programming (DEC-SQP) [23], variable scaling hybrid differential evolution (VSHDE) [24], hybrid differential evolution (DE) [25], bacterial foraging with Nelder–Mead algorithm (BF-NM) [26], hybrid differential evolution with biogeography-based optimization (DE/BBO) [27] etc. have been adopted to solve different types of ELD problems.

Evolutionary algorithms, swarm intelligence, bacterial foraging all are population based bio-inspired algorithm. However, the common disadvantages of these algorithms are complicated computation, using many parameters. Therefore it is difficult to understand these algorithms for beginners. Moreover, all the nature-inspired algorithms such as GA, EP, PSO, ACO, DE, BFA, AIS, BBO etc. require tuning of algorithm parameters for their proper working. Proper selection of parameters is essential for the searching of the optimum solution by these algorithms. A change in the algorithm parameters changes the effectiveness of the algorithms. To avoid this difficulty an optimization method, Teaching–Learning-Based Optimization (TLBO), a parameter free algorithm, is implemented in this paper to solve complex ELD problems.

Teaching–Learning-Based Optimization (TLBO) has been proposed by Rao et al. in the year 2011 **[**28**].** This method works on the effect of influence of a teacher on learners. Like other nature-inspired algorithms, TLBO is also a population based method which uses a population of solutions to proceed to the global solution. For TLBO, the population is considered as a group of learners or a class of learners. The process of working of TLBO is divided into two parts. The first part consists of 'Teacher Phase' and the second part consists of 'Learner Phase'. The 'Teacher Phase' means learning from the teacher and the 'Learner Phase' means learning through the interaction between learners. The teacher is generally considered as a highly learned person who shares his or her knowledge with the learners. The quality of a teacher affects the outcome of learners. It is obvious that a good teacher trains learners such that they can have better results in terms of their marks or grades. Moreover, learners also learn from interaction between themselves, which also helps in their results. Like several other soft computing techniques, TLBO is also a population based technique which implements a group of solutions to proceed for the optimum solution. Many optimization methods require algorithm parameters that affect the performance of the algorithm. Unlike other optimization techniques TLBO does not require any algorithm parameters to be tuned, thus making the implementation of TLBO simpler. TLBO uses the best solution of the iteration to change the existing solution in the population thereby increasing the convergence rate. TLBO uses the mean value of the population to update the solution. Therefore, TLBO implements greediness to accept the good solution. It has been already observed that the performance of TLBO is quite satisfactory when applied to solve continuous benchmark optimization problems [28].

The improved performance of TLBO solve continuous benchmark optimization problems has motivated the present authors to implement this newly developed algorithm to solve different complex ELD problems. This paper considers four types of ELD problems, namely, (i) ELD with quadratic cost function, ramp rate limit, prohibited operating zone and transmission loss: - 15 Generators System, (ii) ELD with quadratic cost function without transmission loss: - 38 Generators System, (iii) ELD with Valve-Point Effects, ramp rate limit, prohibited operating zone: - 140 Generators System, (iv) ELD having Multiple Fuels and Valve-Point Effects: - 160 Generators System.

Section 2 of the paper provides mathematical formulation of different types of ELD problems. Section 3 describes the proposed TLBO algorithm along with a short description of the algorithm used in these test systems. Simulation studies are presented and discussed in Section 4. The conclusion is drawn Section 5.

2. MATHEMATICAL MODELING OF THE ELD PROBLEM

The ELD may be formulated as both convex and non-convex nonlinear constrained optimization problem. Four different types of ELD problems have been formulated and solved by TLBO approach. These are presented below:

2.1 ELD with quadratic cost function, ramp rate limit, prohibited operating zone and transmission loss

The overall objective function F_T of ELD problem may be written as

$$F_T = \min \sum_{i=1}^{N} F_i\left(P_i\right) = \min \sum_{i=1}^{N}\left(a_i + b_i P_i + c_i P_i^2\right) \tag{1}$$

Where, $F_i(P_i)$, is cost function of the i^{th} generator, and is usually expressed as a quadratic polynomial; N is the number of committed generators; a_i, b_i and c_i are the cost coefficients of the i^{th} generator; P_i is the power output of the i^{th} generator. The ELD problem consists in minimizing F_T subject to following constraints:

1) Real Power balance constraint:

$$\sum_{i=1}^{N} P_i - (P_D + P_L) = 0 \tag{2}$$

Where, P_D is the total system active power demand; P_L is the total transmission loss; Calculation of P_L using the *B*- coefficients matrix is expressed as:

$$P_L = \sum_{i=1}^{N}\sum_{j=1}^{N} P_i B_{ij} P_j + \sum_{i=1}^{N} B_{0i} P_i + B_{00} \tag{3}$$

2) The generating capacity constraint:

The power must be generated by each generator within their lower limit $P_i^{\min}$ and upper limit $P_i^{\max}$. So that

$$P_i^{\min} \le P_i \le P_i^{\max} \tag{4}$$

Where $P_i^{\min}$ and $P_i^{\max}$ are the minimum and the maximum power outputs of the i^{th} unit.

3) Ramp Rate Limit Constraint:

The power P_i generated by the i^{th} generator in certain interval neither should exceed that of previous interval P_{io} by more than a certain amount UR_i, the up-ramp limit and nor should it be less than that of the previous interval by more than some amount DR_i, the down-ramp limit of the generator. These give rise to the following constraints:

As generation increases

$$P_i - P_{io} \le UR_i \tag{5}$$

As generation decreases

$$P_{io} - P_i \le DR_i \tag{6}$$

and

$$\max\left(P_i^{\min}, P_{i0} - DR_i\right) \le \min\left(P_i^{\max}, P_{i0} + UR_i\right) \tag{7}$$

4) Prohibited Operating Zone:

Mathematically the feasible operating zones of unit can be described as follows:

$$\begin{aligned} &P_i^{\min} \le P_i \le P_{i,1}^{l} \\ &P_{i,j-1}^{u} \le P_i \le P_{i,j}^{l}; \quad j = 2,3,\ldots n_i \\ &P_{i,n_i}^{u} \le P_i \le P_i^{\max} \end{aligned} \tag{8}$$

Where j represents the number of prohibited operating zones of unit i. $P_{i,j}^{u}$ is the upper limit and $P_{i,j}^{l}$ is the lower limit of the j^{th} prohibited operating zone of i^{th} unit. Total number of prohibited operating zone of the i^{th} unit is n_j.

2.2 ELD with quadratic cost function

In this type of ELD problem the overall objective function is same as mentioned in Equation 1. Here the objective function F_T is to be minimized subject to the constraints of Equation 2, Equation 4. Here P_L is zero.

2.3 ELD with Valve-Point Effects, ramp rate limit, prohibited operating zone

The fuel cost function F_T in ELD problem with valve point loading changes the simple cost function Equation 1. It becomes more complex and is represented below:

$$F_T = (\sum_{i=1}^{N} F_i(P_i)) = (\sum_{i=1}^{N} a_i + b_i P_i + c_i P_i^2 + \left| e_i \times Sin\{ f_i \times (P_i^{\min} - P_i) \} \right|) \tag{9}$$

Where e_i and f_i are the coefficients of the i^{th} generator reflect the valve-point effects. The objective function Equation 9 is to be minimized of subject to the same set of constraints given in Equation 4, Equation 7 and Equation 8.

2.4 ELD with non-smooth Cost Functions with Multiple Fuels and Valve-Point Effects:

For a power system with N generators and n_F fuel options for each unit, the cost function of the generator with valve-point loading is expressed as:

$$\begin{aligned} &F_i(P_i) = a_{ip} + b_{ip} P_i + c_{ip} P_i^2 + \left| e_{ip} \times Sin\{ f_{ip} \times (P_{ip}^{\min} - P_{ip}) \} \right| \\ &if P_{ip}^{\min} \le P_i \le P_{ip}^{\max} \ \ for\ fuel\ option\ p; p = 1, 2, ...n_F \end{aligned} \tag{10}$$

Where, $P_{ip}^{\min}$ and $P_{ip}^{\max}$ are the minimum and maximum power generation limits of i^{th} generator with fuel option p, respectively; $a_{ip}, b_{ip}, c_{ip}, e_{ip}$ and f_{ip} are the fuel-cost coefficients of i^{th} generator for fuel option p.

Considering N numbers of generators, the above-mentioned objective function is to be minimized subject to the constraints of Equation 2, Equation 4, without considering transmission loss. Therefore, P_L term in Equation 2 becomes zero.

2.5 Calculation for slack generator

Let N committed generating units deliver their power output subject to the power balance constraint Equation 2 and the respective capacity constraints of Equation 4 and/or Equation 7, Equation 8. Assuming the power loadings of first (N-1) generators are known, the power level of N^{th} generator (called Slack Generator) is given by

5) *Without Transmission Loss:*

$$P_N = P_D - \sum_{i=1}^{(N-1)} P_i \tag{11}$$

6) *With Transmission Loss:*

$$P_N = P_D + P_L - \sum_{i=1}^{(N-1)} P_i \tag{12}$$

Using Equation 3 and Equation 12, the modified form of equation is:

$$B_{NN}P_N^2 + P_N\left(2\sum_{i=1}^{N-1} B_{Ni}P_i + \sum_{i=1}^{N-1} B_{ON} - 1\right) + \left(PD + \sum_{i=1}^{N-1}\sum_{j=1}^{N-1} P_i B_{ij} P_j + \sum_{i=1}^{N-1} B_{Oi}P_i - \sum_{i=1}^{N-1} P_i + B_{OO}\right) = 0 \tag{13}$$

The solution procedure of Equation 13 to calculate slack generator output, P_N is same as mentioned in [19]. To avoid repetition it is not presented here.

3. TEACHING LEARNING BASED ALGORITHM

This section presents an interesting new optimization algorithm called teaching learning based optimization (TLBO) which has been recently proposed in **[**28]**.** The TLBO method works on the philosophy of the effect of manipulation of a teacher on the output of learners in a class and consequently learners also learn from interaction between themselves, which also helps in their grades. Therefore, the TLBO method works on the philosophy of teaching and learning.

Figure 1

Consider two different teachers, T_1 and T_2, teaching a topic to the same merit level learners in two different classes. The distribution of marks obtained by the learners for this two varying classes evaluated by the teachers is illustrated in Fig. 1. Curves 1 and 2 represent the evaluated marks obtained by the learners taught by teacher T_1 and T_2 respectively. Normal distribution for the goal achieved by learners is defined as:

Figure 2

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{\sigma^2}} \tag{14}$$

where σ^2 is the variance, μ is the mean and x is any value of which normal distribution function is required. Comparing the mean value of curve 1 and curve 2 of Fig. 1, it is seen that the learners from curve 2 get better results than the learners from curve-1. So it can be said that teacher T_2 is better than teacher T_1 in terms of teaching. Learners also learn from interaction between themselves, which promotes their results.

Fig. 2 shows a model for the marks obtained by the learners in a class having mean M_A in curve-A. It is obvious that teacher's as the most intelligence person in the society, therefore the best learner is considered as the teacher here, and this is shown by T_A in Fig.2. The teacher tries to spread the knowledge among the learners which in turn increase the knowledge level of the whole class and help learners to get good marks or grades. Teacher T_A puts maximum effort into teaching his or her students and tries to move class mean from M_A towards a new mean M_B by means of increasing the learners' knowledge level. At that stage the learners require a new teacher T_B, of superior quality than themselves that is shown in curve-B.

TLBO is also a population based algorithm whose population is described as a class of learners. In any nature-inspired based optimization algorithms, the population consists of different design variables. In TLBO, different design variables are the different subjects offered to learners and the learners' outcome is corresponding to the 'fitness function'. The teacher is considered as the best solution obtained so far. The process of TLBO is divided into two parts named 'Teacher Phase' and 'Learner Phase'. The 'Teacher Phase' means learning from the teacher and the 'Learner Phase' means learning through the interaction between learners. Two part of TLBO are described below.

3.1 Teacher phase

A good teacher always tries to improve the quality of learners in terms of knowledge i.e. a teacher tries to increase the mean value of the class from M_A to M_B as in fig. 2. But in real practice this is not possible and a teacher can only move the average quality of a class up to some limit depending on the quality of the class.

Let M_k be the mean and T_k be the teacher at any iteration k. T_k tries to move mean M_k towards its own level. The solution is updated according to the difference between the existing and the new mean. It is given by

$$X_{diff} = rand\,() \mathrm{x} (T_k - R_t\, M_k) \tag{15}$$

where, *rand()* is a random number in the range [0,1]; the value of R_t can be either 1 or 2 which can be decided randomly with equal probability.

This difference modifies the existing solution according to the following expression:

$$X_{new} = X_{old} + X_{diff} \tag{16}$$

3.2 Learner phase

In learner phase, the Learners increase their knowledge in two different methods: first one through input from the teacher and the other through some interaction between themselves. A learner interacts with other randomly selected learners by the participation in formal communications, group discussions and presentations. By the interaction, a learner learns something new if the other learners have more knowledge than the corresponding learner [28]. In order to design the mathematical model, two learners X_i and X_j are randomly chosen, where $i \neq j$. Objective functions for the learners X_i and X_j are evaluated. The achieved objective functions of X_i and X_j are compared. If the achieved objective function of X_i is less than the achieved objective function of X_j , then

$$X_{new} = X_{old} + rand\,() \mathrm{x} (X_i - X_j) \tag{17}$$

Otherwise

$$X_{new} = X_{old} + rand\,() \mathrm{x} (X_j - X_i) \tag{18}$$

If the new solution is better than the existed one then it is accepted. The pseudo codes and flow chart for all steps are available in [28].

3.3 Sequential steps of TLBO algorithm

There are two stages in TLBO: teacher phase and learner phase. All the steps are mentioned below:

1)In initialization stage, read in the initial number of learners (*PopSize*) (equivalent to population size of many heuristic algorithms); maximum iteration number ($Iter_{max}$). Specify no. of design variables

(D), in this case which is assigned as the number of subjects offered. Mention the lower and upper limits of design variables.

2)Generate learner matrix (X_{ij}) randomly according to the population size and the number of design variables and limits of the variables (where i = 1,2,…. $PopSize$, and j = 1,2,………D and total matrix size is ($PopSize \times D$).

3)Determine objective function values for each learner set. Size of the objective function matrix is therefore ($PopSize \times 1$). The minimum value out of these objective function values is local optimum value and the corresponding value of X_{ij} is set as teacher ($X_{teacher}$). So $X_{teacher} = T_k$ in Equation 15.

4)Calculate the mean value of each design variable column wise. So the size of the mean value is ($1 \times D$). The mean value is used in Equation 15 as M_k.

5)Modify each learner by Equation 15 and Equation 16. The value of R_t is randomly selected as 1 or 2. Calculate the objective function values for each modified learner. If the new value of the objective function of any learner is better than the previous one then accept new learner and replace the corresponding old one. Otherwise keep the old learner without any modification.

6) Learner phase: Learners increase their knowledge with the help of their mutual interaction. For each learner X_i (i=1,2,….D), arbitrarily choose any learner X_j from the learner matrix. Compare the objective function corresponding to X_i and X_j. If the value of the objective function of X_i is lower than the objective function value of X_j then modify the i^{th} learner using Equation 17 otherwise modify the i^{th} learner using Equation 18.

7)If the maximum no. of iterations is reached or specified accuracy level is achieved, terminate the iterative process, otherwise go to step 3 for continuation.

Interested readers may refer [28], which contains the detail steps of the TLBO Algorithm.

3.4 TLBO algorithm for economic load dispatch problem

In this subsection, the procedure to implement the TLBO algorithm for solving the ELD problems has been described. This algorithm is also used to deal with the equality and inequality constraints of the ELD problems. The sequential steps of the TLBO algorithm applied to solve ELD problem are:

1. Representation of the learner Matrix X: Since the assessment variables for ELD problem are real power output of the generators, they are together used to represent the individual learner. Each individual element of a learner is the subjects studied by the corresponding learner and it is same as the real power outputs of the generators, in ELD. For initializations choose the number of generator units m as a design variable (*D*) and the total number of learner structure is population size and is denoted as '*PopSize*'.

The complete learner matrix is represented in the form of the following matrix:

$$X = X_i = [X_1, X_2, X_3, ...X_{PopSize}] \; where \;\; i = 1, 2, ..., PopSize$$

In case of ELD problem, each learner is presented as:

$$X_i = [X_{i1}, X_{i2}, ...X_{im}] = [Pg_{ij}] = [Pg_{i1}, Pg_{i2}, ...Pg_{im}];$$

Where, $j = 1, 2, ..., m$. Each learner is one of the possible solutions for the ELD problem. The element X_{ij} of X_i is the j^{th} position component of learner *i*.

2. *Initialization of the learner:* Each individual element of the learner matrix (*X)*, i.e., each element of a given learner is initialized randomly within the effective real power operating limits. The initialization is based on Equation 4 for generators without ramp rate limits, based on Equation 4, Equation 7 for generators with ramp rate limits and based on Equation 4, Equation 7, Equation 8 for generators with ramp rate limits, prohibited operating zone.

3. *Evaluation of objective functions:* In case of ELD problems, objective function, of each learner is represented by the total fuel cost of generation for all the generators of that given learner. It is calculated using Equation 1 for the system having quadratic fuel cost characteristic; using Equation 9 for the system having valve-point effect; using Equation 10 for the system having multi-fuel type fuel cost characteristic.

Now the steps of algorithm to solve ELD problems are given below.

Step 1) For initialization, choose no. of generator units, *m* i.e. no. of design variables (*D*); number of learner, *PopSize*. Specify maximum and minimum capacity of each generator, power demand, B-coefficients matrix for calculation of transmission loss and other input data. Set maximum number of iterations, $Iter_{max}$.

Step 2) Each learner of *X* matrix should satisfy equality constraint of Equation 2 using the concept of slack generator as mentioned in section 2.5.

Step 3) Calculate the objective function value for each learner following the procedure mentioned in "*Evaluation of objective functions*".

Step 4) Based on the objective function values identify the elite learner and that is assigned as teacher of the learner matrix. Here, elite term is used to indicate that learner which gives best fuel cost. The elite learner is taken as T_k in Equation 15.

Step 5) From the learner matrix (X), calculate the mean value of each design variable i.e. the mean value of individual generator's power output column wise. The mean value is assigned as M_k in Equation 15.

Step 6) Modify each learner i.e. power output of the generators using Equation 15 and Equation 16. Verify the feasibility of each newly generated learner of the modified X matrix. Individual element of each modified learner must satisfy the generator operating limit constraint of Equation 4. If any element of a learner violate either upper or lower operating limits, then fix the values of those elements of the corresponding learner at the limit hit by them. Again satisfy constraint of Equation 2 using the concept of slack generator as presented in section 2.5 (P_L=0 in Equation 12 if loss is not considered). If output of slack generator does not meet generator operating limit constraint Equation 4 or some generators do not satisfy the prohibited operating zone or ramp rate limit constraints, where applicable; reject that new learner, and reapply step 6 on its old, until all the constraints are satisfied.

Step 7) Calculate the values of the objective function of each modified learner of the learner matrix. If the new value of the objective function of any learner is better than the previous one then accept new learner and replace the corresponding old one. Otherwise keep the old learner without any modification.

Step 8) For each learner X_i (i=1,2,….D), arbitrarily choose any learner X_j from the learner matrix. Compare the objective function corresponding to X_i and X_j. If the value of the objective function of X_i is lower than the objective function value of X_j then modify the i^{th} learner using Equation 17 otherwise modify the i^{th} learner using Equation 18.

Step 9) Individual element of each modified learner must satisfy their generator constraints. If any element of a modified learner violate either upper or lower operating limits, then fix the values of those elements of the corresponding learner at the limit hit by them. Again satisfy constraint of Equation 2 using the concept of slack generator as presented in section 2.5 (P_L=0 in Equation 12 if loss is not considered). If output of slack generator does not meet generator operating limit constraint Equation 4 or some generators do not satisfy the prohibited operating zone or ramp rate limit constraints, where applicable; reject that modified learner, and reapply step 8 on its old, until all the constraints are satisfied.

Step 10) As individual learners of the learner matrix changes, consequently values of their objective function also changes. Calculate the objective function of each newly generated learner. If the new value of objective function of a given learner is better than its previous value, then accept new learner and replace the corresponding old one. Otherwise keep the old learner without any modification.

Step 11) If the maximum no. of iterations is reached or specified accuracy level is achieved, terminate the iterative process, otherwise go to step 4 for continuation.

4. EXAMPLES AND SIMULATION RESULT

Proposed TLBO algorithm has been applied to solve ELD problems in four different test cases and its performance has been compared to several other optimization techniques like GA [7], DE/BBO [7, 27], and PSO [7, 21] etc. for verifying its feasibility. The necessary codes has been written in MATLAB-7 language and executed on a 2.0-GHz Intel Pentium (R) Dual Core personal computer with 1-GB RAM.

4.1 Description of the Test Systems:-

1) Test System 1: In this example, 15 generating units with ramp rate limit and prohibited zones constraints has been considered. Transmission loss has been included in the problem. Power demand is 2630 MW and system data has been taken from [7]. Results obtained from proposed TLBO, PSO [7] and different versions of PSO [21] and other method have been presented here. Their best solutions are shown in Table 1. The convergence characteristic of the 15-generator systems in case of TLBO is shown in Fig. 3. Minimum, average and maximum fuel costs obtained by TLBO and different versions of PSO [21], over 50 trials are presented in Table 2.

Table 1-2 | Figure 3

2) Test System 2: A 38 generators system with quadratic fuel cost characteristic is used here. The input data are taken in [29]. The load demand is 6000 MW. Transmission loss has not been considered here. The result obtained using proposed TLBO method has been compared with BBO [27], DE/BBO [27], PSO-TVAC [27] and New-PSO [27]. Their best solutions are shown in Table 3. A convergence characteristic of the 38-generator systems in case of TLBO is shown in Fig.4. Minimum, average and maximum fuel costs obtained by TLBO over 50 trials are shown in Table 4.

Table 3-4 | Figure 4

3) Test System 3: A 140 generators system having ramp rate limit, prohibited zones constraints are considered. Effect of valve-point loading has been incorporated within generator fuel cost characteristics of unit no. 5, 10, 15, 22, 33, 40, 52, 70, 72, 84, 119 and 121. The input data of this system are taken from [21]. The load demand is 49342 MW. The best results obtained by proposed TLBO is shown in Table 5. Out of 50 trials, minimum, maximum and average fuel cost obtained using TLBO algorithm, different versions of PSO [21] and Modified Teaching-Learning Algorithm (MTLA) [30] are shown in Table 6. Its convergence characteristic is presented in Fig. 5

Table 5-6 | Figure 5

4) Test System 4: A complex system with 160 thermal units is considered here. The input data are available in [31]. The system demand is 43200 MW. Transmission loss has not been included. The best result obtained using the proposed TLBO algorithm is shown in Table 7. Minimum, average and maximum fuel costs obtained by TLBO, ED-DE [31], and different GA [31] methods over 50 trials are presented in Table 8. Convergence characteristic of the 160-generator systems obtained by TLBO is shown in Fig. 6.

Table 7-8 | Figure 6

4.2 Effect of Learner size for TLBO Algorithms:-

Very large or small value of learner size may not be capable to get the minimum value of fuel cost. For each learner size of 20, 50, 100, 150 and 200, 50 trials has been run. Out of these, learner size of 50 achieves best fuel cost of generations for this system. For other learner size, no significant improvement of fuel cost has been observed. Moreover, beyond learner size = 50, simulation time also increases. Best output obtained by TLBO algorithm for each learner size is presented in Table 9.

4.2.1 Comparative study

Table 9

1. Solution Quality: Tables 1, 3, 5, and 7 present the best fuel cost obtained by TLBO for 4 different test systems. The minimum costs obtained for the 4 test system are better compared to the results obtained by many previously developed techniques are also shown in Tables 2, 4, 6, and 8. These tables also represent the comparative studies for maximum, minimum and average values, obtained by different algorithms. From the results it is clear that the performance of TLBO algorithm is better, in terms of quality of solutions compared to many already existing techniques.

2. Computational Efficiency: In Tables 2, 4, 6 and 8, it is shown that time taken by TLBO to achieve minimum fuel costs, are quite less compared to that obtained by many other techniques. These results prove significantly better computational efficiency of TLBO.

3. Robustness: Performance of any heuristic algorithms cannot be judged by a single run. Normally their performance is judged after running the programs for certain number of trials. Many numbers of trials should be made to obtain a useful conclusion about the performance of the algorithm. An algorithm is said to be robust, if it gives consistent result during these trial runs. Tables 2, 4, 6 and 8 present that out of 50 numbers of trials for four different test systems; TLBO reaches to minimum costs 50, 50, 47 and 47 times respectively. The efficiency of TLBO algorithm to reach minimum solution is 100 % and 94 % respectively. This performance is much superior compared many other algorithms, presented in the different literatures.

Therefore, the above results establish the enhanced ability of TLBO to achieve superior quality solutions, in a computationally efficient and robust way.

5 CONCLUSION

In the present paper, a newly developed TLBO algorithm has been successfully implemented in the field of power system to solve different convex and non-convex ELD problems. The simulation results show that the performance of TLBO is better compared to that of several previously developed optimization techniques. The TLBO has obtained superior quality solutions with high convergence speed in a much robust way. The results also show the advantage of TLBO compared to many previously developed optimization techniques in term of computational time, as the proposed algorithm is parameter free. Therefore, TLBO can be considered as one of the strong tool to solve complex ELD problems. Moreover, successful implementation and superior performance of TLBO to solve ELD problems has created a new path in the field of power system which may encourage the researcher to apply this newly developed algorithm to solve different much complex power system optimization problems like optimal power flow, hydro thermal scheduling, loss minimization, optimal placement of Distributed generators, FACTS devices etc. Therefore, it may finally be concluded that proposed TLBO algorithm is able to solve any complex constrained optimization problems with a faster convergence rate irrespective of the nature of the objective function.

REFERENCES

[1] El-Keib, A.A., Ma, H. and Hart, J. L. "Environmentally constrained economic dispatch using the Lagrangian relaxation method", *IEEE Trans PWRS* 9(4), pp.1723–1729 (1994).

[2] Fanshel, S. and Lynes, E. S., "Economic Power Generation Using Linear Programming", *IEEE Transactions on Power Apparatus and Systems PAS* 83(4), pp.347-356 (1964).

[3] Wood, J. and Wollenberg, B. F., "Power Generation, Operation, and Control", *John Wiley and Sons, 2nd Edition,* (1984).

[4] Walters, D.C. and Sheble, G.B. "Genetic algorithm solution of economic dispatch with valve point loadings", *IEEE Trans PWRS* 8(3), pp.1325-1331 (1993).

[5] Bakirtzis, A. Petridis, V. and Kazarlis, S, Genetic algorithm solution to the economic dispatch problem, Generation transmission and distribution, *IEE proceeding* 141(4), pp.377-382 (1994).

[6] Gaing, Z. L. "Particle swarm optimization to solving the economic dispatch considering the generator constraints", *IEEE Trans PWRS* 18(3), pp.1187-1195 (2003).

[7] Hou, Y.H., Wu, Y.W., Lu, L.J. and Xiong, X.Y. "Generalized Ant Colony Optimization for Economic Dispatch of power systems*", Proceedings of International Conference on Power System Technology Power-Con* 1, pp. 225-229 (2002).

[8] Jayabharathi, T., Jayaprakash, K., Jeyakumar, N., and Raghunathan, T. “Evolutionary programming techniques for different kinds of economic dispatch problems”, *Elect. Power Syst. Res.* 73(2), pp.169-176 (2005).

[9] Panigrahi, C.K., Chattopadhyay, P.K., Chakrabarti, R.N. and Basu, M. “Simulated annealing technique for dynamic economic dispatch”, *Electric power components and systems*, 34(5), pp. 577-586 (2006).

[10] Nomana, N. and Iba, H. “Differential evolution for economic load dispatch problems”, *Elect. Power Syst. Res.* 78(3), pp. 1322–1331 (2008).

[11] Panigrahi, B. K., Yadav, S. R., Agrawal S. and Tiwari, M. K. “A clonal algorithm to solve economic load dispatch”, *Elect. Power Syst. Res.* 77(10), pp. 1381–1389 (2007).

[12] Panigrahi, B. K. and Pandi, V. R. “Bacterial foraging optimization: Nelder-Mead hybrid algorithm for economic load dispatch”, *IET Generation, Transmission, Distribution* 2(4), pp. 556–565 (2008).

[13] Bhattacharya, A. and Chattopadhyay, P.K. “Biogeography-Based Optimization for Different Economic Load Dispatch Problems”, *IEEE Trans PWRS* 25(2), pp. 1064-1077 (2010).

[14] Chiang, C. L. “Improved genetic algorithm for power economic dispatch of units with valve-point effects and multiple fuels”, *IEEE Trans PWRS* 20(4), pp. 1690–1699 (2005).

[15] Alsumait, J.S., Sykulski, J.K. and Al-Othman, A.K. “A hybrid GA–PS–SQP method to solve power system valve-point economic dispatch problems”, *Appl. Energy* 87(5), pp. 1773–1781 (2010).

[16] Sinha, N., Chakrabarti, R. and Chattopadhyay, P.K. “Evolutionary programming techniques for economic load dispatch”, *IEEE Trans. Evol. Comput.* 7(1), pp. 83–94 (2003).

[17] Selvakumar, A.I. and Thanushkodi, K. “A New Particle Swarm Optimization Solution to Nonconvex Economic Dispatch Problems”, *IEEE Trans PWRS* 22(1), pp. 42-51 (2007).

[18] Panigrahi, B.K., Pandi, V.R. and Das, S. “Adaptive particle swarm optimization approach for static and dynamic economic load dispatch”, *Energy Conver. & Managt.* 49(6), pp. 1407–15 (2008).

[19] Chaturvedi, K.T., Pandit, M. and Srivastava, L. "Self-Organizing Hierarchical Particle Swarm Optimization for Nonconvex Economic Dispatch", *IEEE Trans PWRS* 23(3), pp. 1079-1087 (2008).

[20] Vlachogiannis, Vlachogiannis J.K. and Lee, K.Y. "Economic Load Dispatch—A Comparative Study on Heuristic Optimization Techniques With an Improved Coordinated Aggregation-Based PSO", *IEEE Trans PWRS* 24(2), pp. 991-1001(2009).

[21] Park, J.B., Jeong, Y.W., Shin, J.R. and Lee K.Y. "An improved particle swarm optimization for nonconvex economic dispatch problems", *IEEE Trans PWRS* 25(1), pp. 156–166 (2010).

[22] Lu, H., Sriyanyong, P., Song, Y.H. and Dillon, T. "Experimental study of a new hybrid PSO with mutation for economic dispatch with non-smooth cost function", *Int. J. Elect. Power Energy Syst.* 32(9), pp. 921–935 (2010).

[23] Coelho, L. D. S. and Mariani, V. C. "Combining of Chaotic Differential Evolution and Quadratic Programming for Economic Dispatch Optimization With Valve-Point Effect", *IEEE Trans PWRS* 21(2), pp. 989-996 (2006).

[24] Chiou, J. P. "Variable scaling hybrid differential evolution for large-scale economic dispatch problems", *Elect. Power Syst. Res.* 77(3-4), pp. 212–218 (2007).

[25] Duvvuru, N. and Swarup, K. S. "A Hybrid Interior Point Assisted Differential Evolution Algorithm for Economic Dispatch", *IEEE Trans PWRS* 26(2), pp. 541–549 (2011).

[26] Panigrahi, B.K. and Pandi, V. R. "Bacterial foraging optimisation: Nelder–Mead hybrid algorithm for economic load dispatch", *IET Gener. Transm. Distrib.* 2(4), pp. 556–565 (2008).

[27] Bhattacharya, A. and Chattopadhyay, P.K. "Hybrid differential evolution with biogeography-based optimization for solution of economic load dispatch", *IEEE Trans PWRS* 25(4), pp. 1955-1964 (2010).

[28] Rao, R.V., Savsani, V.J. and Vakharia, D.P. "Teaching-learning-based optimization: A noval method for constrained mechanical design optimization problems", *Computer-Aided Design*,43(3), pp. 303-315 (2011).

[29] Yang, H.T., Yang, P.C. and Huang, C.L. "A Parallel Genetic Algorithm Approach to Solving the Unit Commitment Problem: Implementation on the Transputer Networks", *IEEE Trans. Power Syst.*, 12 (2), pp. 661-668 (1997).

[30] Niknam, T., Azizipanah-Abarghooee, R. and Aghaei, J. "A New Modified Teaching-Learning Algorithm for Reserve Constrained Dynamic Economic Dispatch", *IEEE Trans. Power Syst.*, 28(2), pp. 749-763 (2013).

[31] Wang Y., Li, B. and Weise, T. "Estimation of distribution and differential evolution cooperation for large scale economic load dispatch optimization of power systems", *Information Sciences*, 180(12), pp. 2405-2420 (2010).

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Figure Captions

Figure 1:- Marks distribution by learners taught by T_1 and T_2

Figure 2:- Distribution of score for learners

Figure 3:- Convergence characteristic of 15-generators system obtained by TLBO

Figure 4:- Convergence characteristic of 38-generators system obtained by TLBO

Figure 5:- Convergence characteristic of 140-generators system, obtained by TLBO

Figure 6:- Convergence characteristic of 160-generator system, obtained by TLBO

Table Captions

Table 1:- **Best Power Output For 15-Generators System (P_d=2630MW)**

Table 2:- **Comparison Between Different Methods Taken After 50 Trials (15-Generators System)**

Table 3:- **Best Power Output For 38-Generators System (PD=6000MW)**

Table 4:- **Comparison Maximum, Minimum and Average Value Taken After 50 Trials (38-Generators System)**

Table 5:- **Best Power Output For 140-Generators System (PD=49342MW)**

Table 6:- **Comparison Between Different Methods Taken After 50 Trials (140-Generators System)**

Table 7:- **Best Power Output For 160-Generators System (PD=43200MW)**

Table8:- **Comparison between Different Methods Taken After 50 Trials (160-Generators System)**

Table 9:- **Effect of Population Size on 160-Generators System**

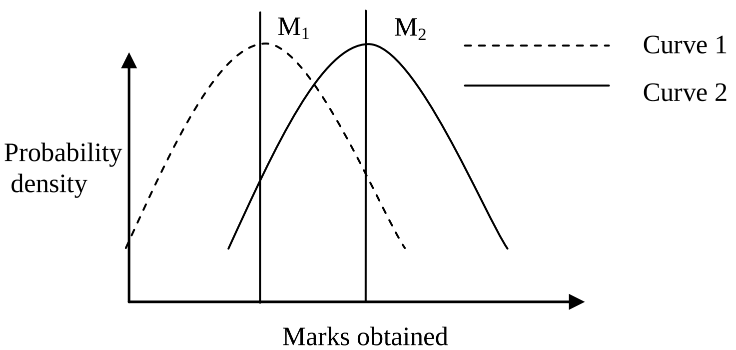
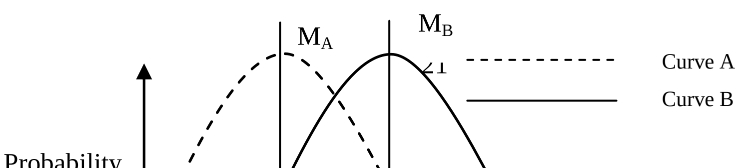


Fig.1. Marks distribution by learners taught by T_1 and T_2



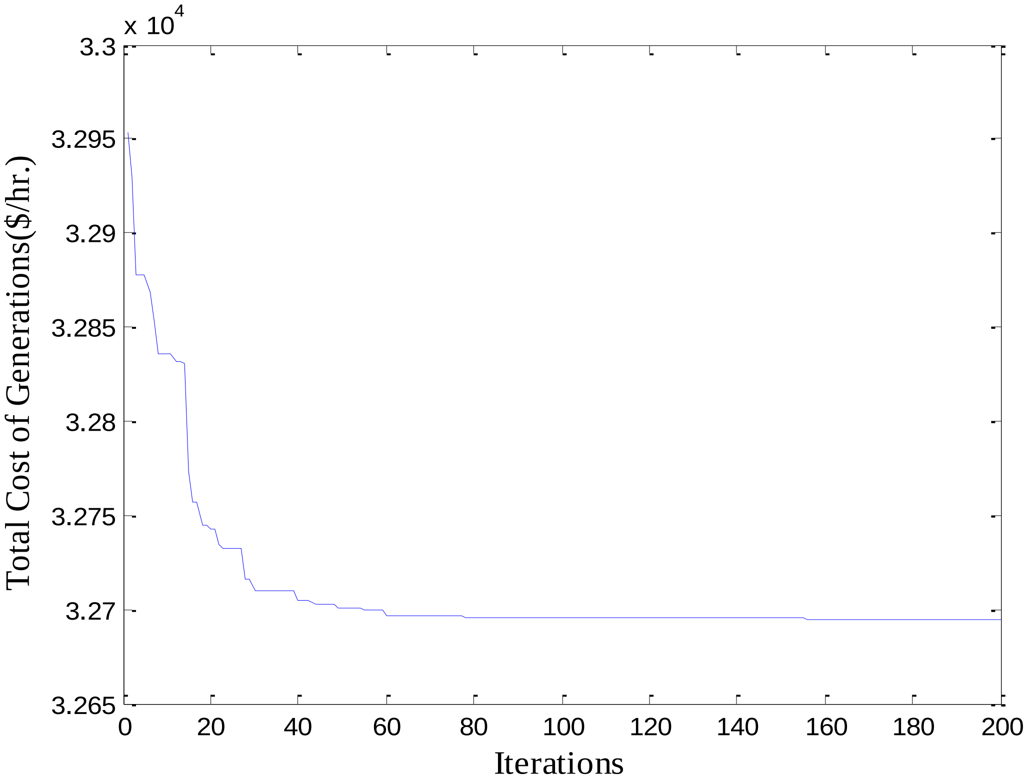


Fig.3. Convergence characteristic of 15-generators system obtained by TLBO

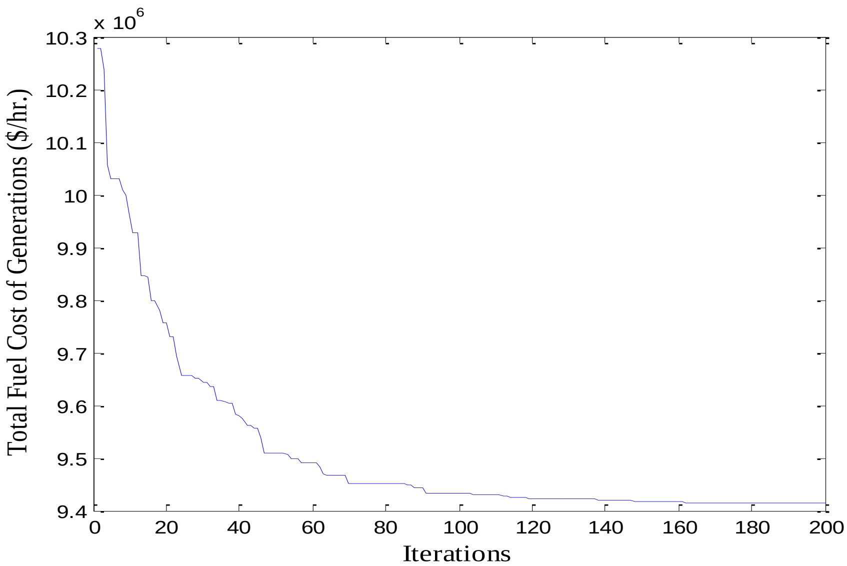


Fig.4. Convergence characteristic of 38-generators system obtained by TLBO

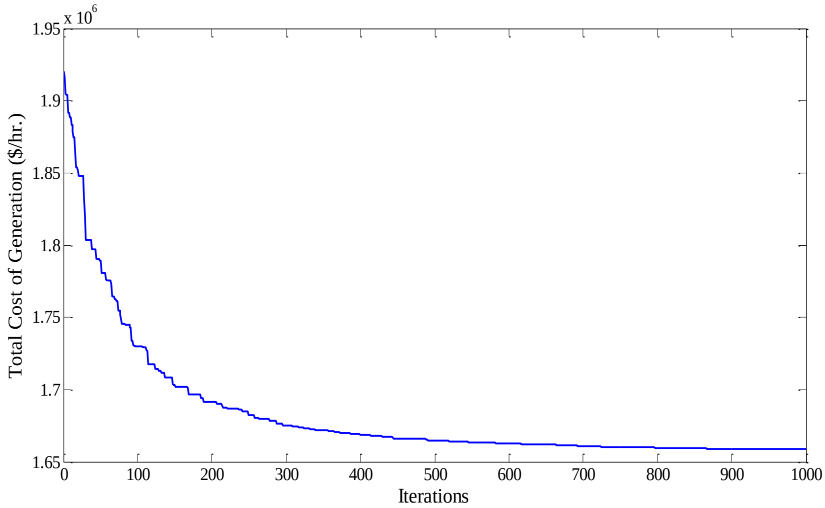


Fig.5. Convergence characteristic of 140-generators system, obtained by TLBO

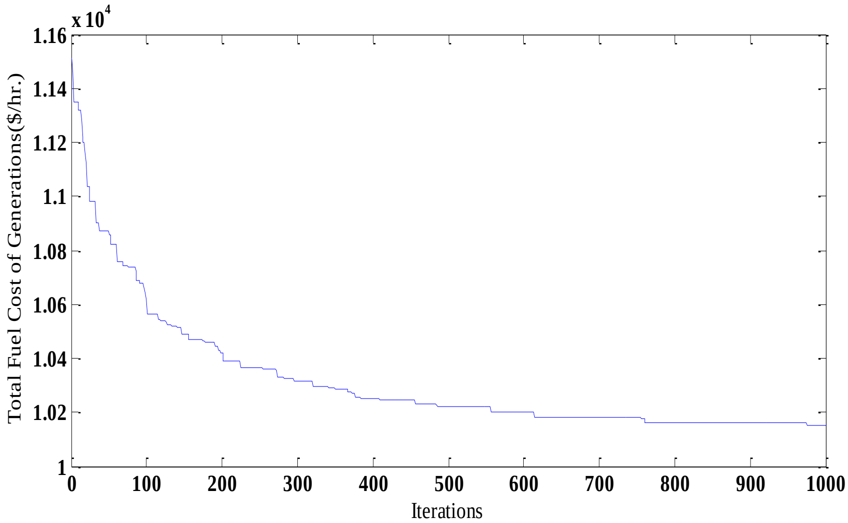


Fig.6. Convergence characteristic of 160-generator system, obtained by TLBO

TABLE 1

BEST POWER OUTPUT FOR 15-GENERATORS SYSTEM (P_D=2630MW)

Unit	TLBO	GA [7]	PSO [7]	CTPSO [21]	CSPSO [21]	COPSO [21]	CCPSO [21]
1	455.000000	415.3108	439.1162	455.0000	455.0000	455.0000	455.0000
2	380.000000	359.7206	407.9727	380.0000	380.0000	380.0000	380.0000
3	130.000000	104.4250	119.6324	130.0000	130.0000	130.0000	130.0000
4	130.000000	74.9853	129.9925	130.0000	130.0000	130.0000	130.0000
5	170.000000	380.2844	151.0681	170.0000	170.0000	170.0000	170.0000
6	460.00000	426.7902	459.9978	460.0000	460.0000	460.0000	460.0000
7	430.000000	341.3164	425.5601	430.0000	430.0000	430.0000	430.0000
8	73.081166	124.7867	98.5699	71.7430	71.7408	71.7427	71.7526
9	51.646599	133.1445	113.4936	58.9186	58.9207	58.9189	58.9090
10	160.000000	89.2567	101.1142	160.0000	160.0000	160.0000	160.0000

11	80.000000	60.0572	33.9116	80.0000	80.0000	80.0000	80.0000
12	80.000000	49.9998	79.9583	80.0000	80.0000	80.0000	80.0000
13	26.577183	38.7713	25.0042	25.0000	25.0000	25.0000	25.0000
14	17.150894	41.9425	41.4140	15.0000	15.0000	15.0000	15.0000
15	16.033243	22.6445	35.6140	15.0000	15.0000	15.0000	15.0000
Total (MW)	2659.489085	2668.4	2662.4	2660.6615	2660.6615	2660.6615	2660.6616
Loss (MW)	29.489085	38.2782	32.4306	30.6615	30.6615	30.6615	30.6616
Fuel Cost ($/hr.)	**32697.215085**	33113	32858	32704	32704	32704	32704

TABLE 2

COMPARISON BETWEEN DIFFERENT METHODS TAKEN AFTER 50 TRIALS

(15-GENERATORS SYSTEM)

Methods	Generation Cost ($/hr.)				Time/Iteration (Sec)	No. of hits to minimum solution
	Max.	Min.	Average	Standard Deviation		
TLBO	**32697.215085**	**32697.215085**	**32697.215085**	**0.00**	**4.0**	**50**
CTPSO[21]	32704.4514	32704.4514	32704.4514	-	22.5	NA*
CSPSO[21]	32704.4514	32704.4514	32704.4514	-	16.1	NA
COPSO[21]	32704.4514	32704.4514	32704.4514	-	85.1	NA
CCPSO[21]	32704.4514	32704.4514	32704.4514	-	16.2	NA

* NA:- Data Not Available

TABLE 3

BEST POWER OUTPUT FOR 38-GENERATORS SYSTEM (P_D=6000MW)

Output	TLBO	DE/BBO[27]	BBO[27]	PSO_TVAC[27]	NEW_PSO[27]
P_1	425.891375	426.606060	422.230586	443.659	550.000
P_2	426.828618	426.606054	422.117933	342.956	512.263
P_3	430.318693	429.663164	435.779411	433.117	485.733
P_4	429.480487	429.663181	445.481950	500.00	391.083
P_5	429.996241	429.663193	428.475752	410.539	443.846
P_6	430.036039	429.663164	428.649254	492.864	358.398
P_7	429.142948	429.663185	428.119288	409.483	415.729
P_8	428.764849	429.663168	429.900663	446.079	320.816
P_9	114.000000	114.000000	115.904947	119.566	115.347
P_{10}	114.000000	114.000000	114.115368	137.274	204.422
P_{11}	119.373112	119.768032	115.418662	138.933	114.000
P_{12}	127.864848	127.072817	127.511404	155.401	249.197
P_{13}	110.000000	110.000000	110.000948	121.719	118.886
P_{14}	90.000000	90.0000000	90.0217671	90.924	102.802
P_{15}	82.000000	82.0000000	82.0000000	97.941	89.0390
P_{16}	120.000000	120.000000	120.038496	128.106	120.000
P_{17}	159.332636	159.598036	160.303835	189.108	156.562

P_{18}	65.000000	65.0000000	65.0001141	65.0000	84.265
P_{19}	65.000000	65.0000000	65.0001370	65.0000	65.041
P_{20}	271.994045	272.000000	271.999591	267.422	151.104
P_{21}	271.999334	272.000000	271.872680	221.383	226.344
P_{22}	259.997110	260.000000	259.732054	130.804	209.298
P_{23}	130.995978	130.648618	125.993076	124.269	85.719
P_{24}	10.000001	10.0000000	10.4134771	11.535	10.000
P_{25}	113.306372	113.305034	109.417723	77.103	60.000
P_{26}	88.045293	88.0669159	89.3772664	55.018	90.489
P_{27}	37.532207	37.5051018	36.4110655	75.000	39.670
P_{28}	20.000000	20.0000000	20.0098880	21.628	20.000
P_{29}	20.000000	20.0000000	20.0089554	29.829	20.995
P_{30}	20.000000	20.0000000	20.0000000	20.326	22.810
P_{31}	20.000000	20.0000000	20.0000000	20.000	20.000
P_{32}	20.000000	20.0000000	20.0033959	21.840	20.416
P_{33}	25.000000	25.0000000	25.0066586	25.620	25.000
P_{34}	18.000000	18.0000000	18.0222107	24.261	21.319
P_{35}	8.000000	8.00000000	8.00004260	9.6670	9.1220
P_{36}	25.000000	25.0000000	25.0060660	25.000	25.184
P_{37}	21.907418	21.7820891	22.0005641	31.642	20.000
P_{38}	21.192396	21.0621792	20.6076309	29.935	25.104
Fuel Cost($/hr.)	**9411938.5572307333**	9417235.786391673	9417633.6376443729	9500448.307	9516448.312

TABLE 4

COMPARISON MAXIMUM, MINIMUM AND AVERAGE VALUE TAKEN AFTER 50 TRIALS

(38-GENERATORS SYSTEM)

Methods	Generation Cost ($/hr.)				Time/Iteration (Sec)	No. of hits to Min. solution
	Max.	Min.	Average	Standard Deviation		
TLBO	9411938.55723073	9411938.55723073	9411938.55723073	0.00	0.50	50

TABLE 5

BEST POWER OUTPUT FOR 140-GENERATORS SYSTEM (PD=49342MW)

Unit	Power Output(MW)	Unit	Power	Unit	Power Output(MW)
P_1	119.000000	P_{48}	249.994057	P_{95}	837.500000
P_2	163.992556	P_{49}	249.946191	P_{96}	682.000000
P_3	189.972341	P_{50}	249.929215	P_{97}	720.000000
P_4	189.998972	P_{51}	165.209529	P_{98}	718.000000
P_5	168.535362	P_{52}	165.011169	P_{99}	720.000000
P_6	189.997956	P_{53}	165.016223	P_{100}	964.000000
P_7	490.000000	P_{54}	165.451209	P_{101}	958.000000
P_8	490.000000	P_{55}	180.017382	P_{102}	947.900000
P_9	496.000000	P_{56}	180.022796	P_{103}	934.000000
P_{10}	496.000000	P_{57}	103.221141	P_{104}	935.000000
P_{11}	496.000000	P_{58}	198.019702	P_{105}	876.500000

P_{12}	496.000000	P_{59}	312.000000	P_{106}	880.900000
P_{13}	506.000000	P_{60}	310.335980	P_{107}	873.700000
P_{14}	509.000000	P_{61}	163.059478	P_{108}	877.400000
P_{15}	506.000000	P_{62}	95.011962	P_{109}	871.700000
P_{16}	505.000000	P_{63}	510.936198	P_{110}	864.800000
P_{17}	506.000000	P_{64}	510.798512	P_{111}	882.000000
P_{18}	506.000000	P_{65}	489.960051	P_{112}	94.008366
P_{19}	505.000000	P_{66}	255.973389	P_{113}	94.008341
P_{20}	505.000000	P_{67}	489.682262	P_{114}	94.002109
P_{21}	505.000000	P_{68}	490.000000	P_{115}	244.043393
P_{22}	505.000000	P_{69}	130.012045	P_{116}	244.017301
P_{23}	505.000000	P_{70}	339.411380	P_{117}	244.021535
P_{24}	505.000000	P_{71}	139.530668	P_{118}	95.016467
P_{25}	537.000000	P_{72}	388.321434	P_{119}	95.012018
P_{26}	537.000000	P_{73}	201.593238	P_{120}	116.010750
P_{27}	549.000000	P_{74}	175.736242	P_{121}	175.016446
P_{28}	549.000000	P_{75}	211.418208	P_{122}	2.000193
P_{29}	501.000000	P_{76}	274.267672	P_{123}	4.001186
P_{30}	499.000000	P_{77}	382.327348	P_{124}	15.012599
P_{31}	506.000000	P_{78}	330.234153	P_{125}	9.010491
P_{32}	506.000000	P_{79}	531.000000	P_{126}	12.001651
P_{33}	506.000000	P_{80}	531.000000	P_{127}	10.001491
P_{34}	506.000000	P_{81}	541.971416	P_{128}	112.019297
P_{35}	500.000000	P_{82}	56.003078	P_{129}	4.004812
P_{36}	500.000000	P_{83}	115.032582	P_{130}	5.034679
P_{37}	241.000000	P_{84}	115.003931	P_{131}	5.001229
P_{38}	241.000000	P_{85}	115.027600	P_{132}	50.000415
P_{39}	774.000000	P_{86}	207.012109	P_{133}	5.001042
P_{40}	769.000000	P_{87}	207.012532	P_{134}	42.021338
P_{41}	3.014093	P_{88}	175.000656	P_{135}	42.002799
P_{42}	3.001595	P_{89}	175.148390	P_{136}	41.005287
P_{43}	250.000000	P_{90}	182.053148	P_{137}	17.004924
P_{44}	249.166734	P_{91}	175.129746	P_{138}	7.018298
P_{45}	250.000000	P_{92}	575.400000	P_{139}	7.001898
P_{46}	249.803132	P_{93}	547.500000	P_{140}	26.291702
P_{47}	249.981180	P_{94}	836.800000	Cost ($/hr.):- **1657586.7157401750**	

TABLE 6

COMPARISON BETWEEN DIFFERENT METHODS TAKEN AFTER 50 TRIALS

(140-GENERATORS SYSTEM)

Methods	Generation Cost ($/hr.)				Time/Iteration (Sec)	No. of hits to Min. Solution
	Max.	Min.	Average	Standard Deviation		
TLBO	1657596.2512	1657586.7157	1657587.2878	2.2875	12.8	47
CTPSO[21]	1658002.7900	1657962.7300	1657964.0600	-	100	NA
CSPSO[21]	1657962.8500	1657962.7300	1657962.7400	-	99	NA
COPSO[21]	1657962.7300	1657962.7300	1657962.7300	-	150	NA
CCPSO[21]	1657962.7300	1657962.7300	1657962.7300	-	150	NA
MTLA[30]	1657951.9053	1657951.9053	1657951.9053	-	2.28	NA

TABLE 7

BEST POWER OUTPUT FOR 160-GENERATORS SYSTEM (P_D=43200MW)

Unit	Power Output(MW)	Unit	Power Output(MW)	Unit	Power Output(MW)
P_1	230.231072	P_{55}	268.702738	P_{109}	420.285071
P_2	210.446980	P_{56}	235.502986	P_{110}	274.018137
P_3	286.038870	P_{57}	299.035088	P_{111}	223.101446
P_4	242.966508	P_{58}	243.769370	P_{112}	211.644783
P_5	282.709600	P_{59}	435.661539	P_{113}	287.251212
P_6	241.572630	P_{60}	275.710290	P_{114}	237.024233
P_7	293.534605	P_{61}	236.103212	P_{115}	276.677853
P_8	241.844669	P_{62}	211.015683	P_{116}	242.713774
P_9	428.520764	P_{63}	269.917872	P_{117}	298.850305
P_{10}	273.970461	P_{64}	240.104662	P_{118}	240.329015
P_{11}	223.822498	P_{65}	289.996904	P_{119}	409.123323
P_{12}	213.659852	P_{66}	247.063654	P_{120}	268.489742
P_{13}	296.793002	P_{67}	297.981577	P_{121}	216.378669
P_{14}	243.099952	P_{68}	235.432930	P_{122}	223.185522
P_{15}	283.978652	P_{69}	436.397367	P_{123}	282.345249
P_{16}	241.597356	P_{70}	273.077310	P_{124}	244.262927
P_{17}	284.202003	P_{71}	231.023042	P_{125}	278.861687
P_{18}	243.441124	P_{72}	211.442586	P_{126}	242.752663
P_{19}	430.831438	P_{73}	263.863789	P_{127}	274.027211
P_{20}	283.002119	P_{74}	245.211579	P_{128}	240.527749
P_{21}	217.450883	P_{75}	262.494238	P_{129}	436.881594
P_{22}	213.075368	P_{76}	237.375342	P_{130}	275.124501
P_{23}	279.454877	P_{77}	278.695112	P_{131}	222.507071
P_{24}	238.652449	P_{78}	243.530392	P_{132}	210.184492
P_{25}	267.130395	P_{79}	438.426795	P_{133}	279.589430
P_{26}	238.526165	P_{80}	270.445893	P_{134}	232.842543
P_{27}	274.066249	P_{81}	221.562195	P_{135}	274.300277
P_{28}	242.075343	P_{82}	210.474538	P_{136}	235.855180
P_{29}	427.901473	P_{83}	293.338588	P_{137}	291.097887
P_{30}	264.284943	P_{84}	241.945638	P_{138}	236.748676
P_{31}	219.466474	P_{85}	301.572104	P_{139}	435.188836
P_{32}	209.112710	P_{86}	241.132843	P_{140}	258.139680
P_{33}	287.864658	P_{87}	289.654387	P_{141}	203.969339
P_{34}	241.574369	P_{88}	234.692550	P_{142}	208.977942
P_{35}	272.641652	P_{89}	431.272142	P_{143}	283.658807
P_{36}	234.826416	P_{90}	273.957457	P_{144}	238.575237
P_{37}	292.822639	P_{91}	219.095369	P_{145}	280.256373
P_{38}	237.978690	P_{92}	214.723938	P_{146}	241.034880
P_{39}	436.636667	P_{93}	283.451750	P_{147}	289.328078
P_{40}	265.432210	P_{94}	245.506570	P_{148}	241.582038
P_{41}	217.930519	P_{95}	273.004206	P_{149}	432.032684
P_{42}	222.583499	P_{96}	236.794502	P_{150}	273.777239
P_{43}	290.494600	P_{97}	291.482917	P_{151}	216.757453
P_{44}	233.438274	P_{98}	235.155228	P_{152}	225.888284
P_{45}	295.299022	P_{99}	416.363970	P_{153}	271.727563
P_{46}	237.854959	P_{100}	255.410343	P_{154}	234.249233
P_{47}	278.510221	P_{101}	216.223183	P_{155}	276.486019

P_{48}	248.035703	P_{102}	209.982441	P_{156}	236.439473
P_{49}	424.865371	P_{103}	256.746663	P_{157}	281.837647
P_{50}	275.625373	P_{104}	238.692105	P_{158}	238.199988
P_{51}	211.497807	P_{105}	276.972440	P_{159}	438.866929
P_{52}	205.196578	P_{106}	241.383638	P_{160}	267.589374
P_{53}	284.858260	P_{107}	270.763647	Cost ($/hr.)	**10005.9944539382**
P_{54}	236.131977	P_{108}	239.556436		

TABLE 8

COMPARISON BETWEEN DIFFERENT METHODS TAKEN AFTER 50 TRIALS

(160-GENERATORS SYSTEM)

Methods	Generation Cost ($/hr.)				Time/Iteration (Sec)	No. of hits to Min. Solution
	Max.	Min.	Average	Standard Deviation		
TLBO	10006.28210000	10005.9944539382	10006.01170000	0.0690	48.216	47
ED-DE[31]	NA	10012.68	NA	-	NA	NA
CGA-MU[31]	NA	10143.73	NA	-	NA	NA
IGA-MU[31]	NA	10042.47	NA	-	NA	NA

TABLE 9

EFFECT OF LEARNER SIZE ON 160-GENERATORS SYSTEM

Learner size	No. of hits to Best Solution	Simulation Time(Sec.)	Max. Cost ($/hr.)	Min. Cost ($/hr.)	Average Cost ($/hr.)
20	23	47.765	10006.8320	10006.5210	10006.6890
50	**47**	**48.216**	**10006.2821**	**10005.9944**	**10006.0117**
100	20	53.233	10006.7609	10006.5274	10006.6675
150	12	58.610	10006.9919	10006.5751	10006.8919
200	10	64.702	10007.2527	10006.5962	10007.1214