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RESEARCH ARTICLE

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Design and multiobjective optimization of a two-point contact ladder-climbing robot using a genetic algorithm

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Abstract

This paper presents the design and optimization of a climbing robot. The design of a ladder-climbing robot is done with fundamental mathematical considerations. The designed robot is robust enough to manage all environmental calamities, and at the same time, it is optimized for lightweight to reduce the actuator's cost and ease of transportation. An analytical evaluation is carried out for both static and dynamic conditions to determine strength and motion characteristics. The multiobjective optimization of the design parameters of a ladder-climbing robot is done to obtain optimized values of design parameters. The formulation of an optimization problem that considers the minimization of weight and natural frequency is performed. Using an evolutionary genetic algorithm (GA) for the multicriteria optimization problem is solved, and a Pareto front solution is obtained. The optimal values of the parameters are decided based on the knee selection technique. As both objective functions are contradictory, the optimum results significantly improve the robot's performance. Controlling the proportional–integral–derivative (PID) parameters is crucial as the robot climbs with a two-point contact gait pattern. The controlling parameters impart stability to the robot. PID parameters like proportional, integral and derivative gain are tunned using the GA. Finally, the developed prototype is tested on the ladders of the tower, and satisfactory climbing motion is achieved.

KEYWORDS

climbing mechanism, genetic algorithm, maintenance, multiobjective optimization, Pareto front, PID, robotics, stability analysis, telecom tower

1 | INTRODUCTION

Designing a robot based on applications is a common practice that engineers and scientists are adopting nowadays. A ladder-climbing robot for telecom tower maintenance and inspection can bring revolutionary changes in the tower maintenance field. The present work aims to conceptualize a robot's design that can climb a ladder and inspect a telecom tower. The static and dynamic analysis followed by optimization of the design and control parameters of the proposed robot for its performance improvement is performed, and the robot prototype is developed to climb a vertical ladder.

The maintenance of widely spread telecommunication network towers is a big problem that telcos face, as they need constant connectivity. Telecom network tower maintenance and inspection are crucial as the towers are located at each corner of the world, and demand for connectivity increases tremendously. Telecommunication companies in Europe, Asia and the United States face higher population density (Analysis, Global Market, 2019; Baroudy et al., 2023; Economy, 2021; Stanley, 2019). Robotics implementation for challenging working situations is a green technological solution. The fast-growing field of telecommunication and higher connectivity demand has made the task of tower maintenance more difficult and pressurized (Jang et al., 2018; Lirov & Yue, 1991; Rosu et al., 2018; Telecom, 2015). Most of the tower maintenance work involves maintaining equipment installed at the top of the tower. A ladder attached to the tower is required to climb to access the devices on the tower (Shah & Dave, 2023). The riggers (Certified Tower Climbers) perform maintenance duties on top of the tower, heights ranging from 25 to 80 m. It is found that they become victimized by work-at-height hazards. The possible work-at-height hazards include bird hits, falls from the tower, electric shocks, insect bites, cyclonic wind and heavy rain (Banta & Banta, 1988; Challenges, 2018; Latif et al., 2017; Maintenance, 2020; Safety, 2018). As per the Occupational Safety and Health Administration's (OSHA, 2022) record, the fatality rate in telecom maintenance is 10 times higher than that in construction jobs. The scarcity of skilled workforce and network spread in each corner of the world fuel the burning issue (Indian Telecom Market, 2020; Information, 2019; Market, 2020; OSHA Report Part 2, 2020).

The authors interviewed telecom veterans and collected their opinions on telcos' challenges in operating and maintaining network towers. They agreed that automation through robotics could be the need of the day and future. They even suggested the silent features of the automatic system desired. The automation system has features like compact design, robust system, data collection and storage, and ease of transportation from one site to another. It can also assist the rigger in transferring tools and equipment from the bottom to the top of the tower. On the basis of their suggestion and opinions, implementing automation through robotics seems to be a good solution (Shah & Dave, 2023). A robot capable of climbing the ladders attached to the telecom towers can perform the inspection with the camera attached. The robot collects and stores the primary data that can be helpful to the rigger. The riggers carry tool bags and equipment weighing nearly 3–4 kg while climbing the tower. A robot designed for a higher payload can work as a tool carrier to transfer loads from the bottom to the top of the tower and assist the rigger.

Authors have conducted a bidirectional literature survey to impart feasible solutions to the problem. One direction is telecom network tower types, their height variants and climbing arrangements, and the other direction is in the area of robotics and automation to judge the capabilities of current developments in climbing robots to be utilized. An exhaustive literature survey is conducted to understand the mechanism of existing climbing robots in research and development. Climbing robots are becoming more popular because of their versatile applications. Literature related to slopes, stairs, inclined and curved surfaces, and wall climbing can provide a solid base for design mechanisms for ladder climbing (Shah et al., 2022). Automated platforms for oblique manipulation, snake robots to climb pipes and structure climber robots may support in designing the mechanism (Blatnický et al., 2020; Virgala et al., 2020). The climbing robot presented by Peidró et al. (2019) is a two-jaw gripper pipe climbing robot. Pipe, pole, structure, uneven surfaces climbing robots consist of wheeled, gripping jaws, magnetic or vacuum adhesion system. Peidró et al. (2019) developed a robot to climb three-dimensional (3D) structures for pipe inspection. Omoto et al. (2019) proposed the optimization of humanoid climbers for locomotion systems. Liu et al. (2018) presented a dry adhesive linkage device to climb uneven vertical surfaces. Robots use magnetic grippers, vacuum grippers and linkages to climb trees and structures (Shah, Dave, Detharia, et al., 2021; Shah, Dave, Majithiya, et al., 2021; Shah et al., 2022). Designs of a climbing robot with claws, parallel robots, soft multileg, bioinspired, with climbing limbs are available in the research (Asalekar et al., 2017; Hao et al., 2011; Hatoum, 2018; Ryu et al., 2010; Sakuhara et al., 2020; Singh, 2015). Climbing robot manufacturers have developed pipe, pole, wall and surface climber machines (International Climbing Machines, 2020; Naugra, 2021; Robotics, 2020; Robots, 2019). Critically examining the literature, it is seen that in climbing robots, almost all the available robots are developed for applications (Buchanan et al., 2021; Gerdes et al., 2020; Kolvenbach et al., 2020; Tremblay et al., 2020). Telecommunication network tower maintenance is done manually by a crew of riggers worldwide. To the best of the authors' knowledge, developing a robot to climb on a telecom tower and perform inspection tasks has not been studied and implemented to date.

The design of a ladder-climbing robot to inspect telecom network towers has to satisfy several criteria. It can withstand all kinds of environmental situations like heavy cyclonic wind and rain and does not conflict with the radiation frequency of the tower itself. The designed robot is optimized for lightweight, which reduces the actuator's cost. Optimization is performed to develop an economically viable robot with improved performance. Optimization of the design parameters is helpful in meeting performance requirements. The optimization criteria selection is an important parameter considering the robot's payload, speed of travel, and strength of the robot. Lightweight robots without compromising strength and minimization of natural frequency are the two optimization criteria. Reduction in weight and natural frequency both are contradictory objectives. The design parameters of the ladder-climbing robot are optimized by considering multiobjective optimization (MOO) for the minimization of weight and natural frequency. MOO problems solution does not result in a single solution for parameter values. However, it results in a range of solutions in the form of a Pareto front. The knee decision selection technique is used to get the optimized parameter values.

Many optimization techniques are available to determine a problem's desired solution. Chablat et al. (2018) proposed mechanical design optimization of a pipe inspection robot. Hassan and Abomoharam (2017) used nondominated sorting genetic algorithm version 2 (NSGA-II) for the robot gripper mechanism. Researchers are using genetic algorithm (GA) techniques for application-based MOO for industrial and service robotic manipulators is a practice researchers are using to get optimum results. For optimizing the gripper design of a robot GA-based MOO solution, NSGA-II is adopted to establish relationships between the objectives. Specific techniques like multiobjective Grey Wolf Optimizer methodology for single mobile robot motion optimization are available (Elsisi, 2024; Petrović et al., 2022), and using reference-point-based nondominated sorting approach (Deb & Jain, 2014). GA suits well for optimization for the machining process, IOT bases supply chain management and multicriteria decision-making modal development (Čuboňová et al., 2019; Muthuswamy & Ali, 2023; Nabeeh, 2023; Sallam et al., 2023). In the problems of autonomous navigation and control optimization plays a vital role (Elsisi et al., 2021, 2023; Urban et al., 2020). Artificial intelligence-based techniques for the optimization of specific applications are the current trends in research in the area of optimization (Agajie et al., 2023; Bergies et al., 2022; Essa et al., 2022; Mohamed et al., 2022).

One of the most important aspects is the formulation of the optimization problem. The results of the optimizations heavily depend on the formulation of the problem. The inaccurate formulation may lead to unacceptable solutions. Therefore, it is essential to formulate the problem and optimality criteria carefully. A GA technique is proper for solving two contradictory objectives and optimization problems, resulting in global minima with the fastest convergence rate. A few research papers and patents highlight the MOO of the robot designed for specific issues (Bhoskar et al., 2015; Shah et al., 2024b; Kouritem et al., 2022; Zeid et al., 2024).

A proportional–integral–derivative (PID) controller controls motion and imparts dynamic stability to a robot in all situations. It is necessary to develop the system's transfer function with primary mathematical considerations. The stability of the robot depends on the PID parameter values. It is required to adopt precise values of proportional (k_p), integral (k_i) and derivative (k_d) gains to ensure dynamic stability. Tuning PID parameters is a tedious task. A GA is the most suitable technique to improve the solution quality (Alvarez-Ramirez et al., 2000; Martins, 2005; Wen & Murphy, 1990). A GA is used to optimize the values of PID parameters, and a smooth response without overshoot is achieved.

There are many other methods to solve such multicriteria problems. Traditional techniques like Gradient Descent and Linear Programming have limitations in dealing with the nonlinearity of robot optimization problems (Bhoskar et al., 2015). Other evolutionary algorithms like particle swarm optimization (PSO) might be struck in local minima, and differential evolution may need careful parameter tuning (Sinha et al., 2018). Recent advancements in Reinforcement Learning have demonstrated its potential in robotic control (Jiang et al., 2020). However, the high computational cost and the need for extensive training environments can be drawbacks compared with GAs. The proposed GA method for the design and MOO of a two-point contact ladder-climbing robot lies in its robustness, flexibility and ability to handle complex, nonlinear and multimodal optimization problems. Its global search capability, adaptability to multiple objectives, and practical ease of implementation make it an excellent choice for this application compared with traditional optimization methods, other evolutionary algorithms and machine learning-based approaches.

Looking at the need for the development of a climbing robot and applying the optimization, the present work's objectives are as follows: (1) to conceptualize the design of a robotic mechanism capable of climbing a telecommunication network tower's ladder, (2) to perform static and dynamic analysis to obtain motion characteristics, (3) to carry out MOO to minimize weight and natural vibration frequency and optimize the PID controller parameters and (4) to develop a prototype that climbs the ladder of the telecom tower.

The rest of the paper is organized according to the conceptual design, analysis and optimization sequence. The detailed methodology adopted for a solution is mentioned in Section 2. Section 3 includes the proposed robot's detailed mechanical design and static and dynamic analysis. The MOO for the design parameters using a GA is presented in Section 4. Appendix 1 includes the results of the GA for design parameters optimization. Tuning PID parameters is required for the robot to move smoothly on the ladder. Optimization of PID parameter optimization is also presented in Section 4. The robot prototype is developed, and its trial is taken on the ladder. Section 5 contains the development and motion details of the prototype. The concluding remarks are presented in Section 6.

2 | METHODOLOGY

Robotic application for the automation of field maintenance of telecom towers requires developing a robust engineering automation system consisting of compact, technologically sound, economic and environment-friendly robots. There are higher chances of dealing with challenges and issues while developing such a multicriteria system. The first step is to conceptualize the design of a robot that is capable of climbing the ladders attached to the telecom tower. The next step is to develop a computer-aided design (CAD) model motion simulation and perform static and dynamic analysis of the robot. To fulfil versatile issues, a designer has to optimize system parameters on different fronts. Optimization of the robot is done for design parameters and controlling parameters. A MOO problem is formulated, considering technical, social, economic and environmental issues on a large and interconnected scale. The ladder-climbing robot has to climb a ladder height ranging from 25 to 80 m with a high play load. MOO is performed to optimize design parameters using a GA. The optimization of the controlling parameters is performed to improve performance and develop a cost-effective solution. For design parameter optimization, one of the criteria to be optimized is the weight reduction of the robot without compromising strength; the other criterion is the minimization of the system's natural frequency. The objective functions and constrained equations are formulated based on these criteria. The optimization with MOGA results in Pareto front of sets of nondominated sorting points sets using NSGA-II for the objective functions. The Pareto front shows multiple optimal solutions that fit into the objective functions. The decision on the optimum values is highly important to arrive at the solution. The knee selection technique is utilized to identify the optimum result and optimum value of design variables obtained. The solution to MOO problems is highly challenging while resolving conflict between objective functions without compromising the other objectives. It is required to trade-off between the multiple optimal solutions (Bhoskar et al., 2015; Deb, 1995; Deb & Jain, 2014; Kouritem et al., 2022; Petrović et al., 2022; Srinivas & Deb, 1994).

To optimize the controlling parameters of the system PID controller is tuned to arrive at the values of proportional (k_p), derivative (k_d) and integral (k_i) gain. The optimization of PID parameters is done using a GA. To optimize controlling parameters integral time absolute error (ITAE) is minimized and an overshoot-free system response curve is obtained. Optimized values of proportional (k_p), derivative (k_d) and integral (k_i) gain were utilized to simulate the dynamic response of the system. Figure 1 shows the detailed methodology for solving the robot design and optimization problem.

A GA is chosen in the present work by looking at the importance of formulating an optimization problem to avoid inaccuracy and unacceptable solutions. A GA is an optimization method based on the fundamentals of natural selection of the process parameters. A multiple-pass algorithm searches many peaks in parallel, eliminating the chances of trapping into local minima. Therefore, it is a suitable technique for moo to reach the fastest convergence (Bhoskar et al., 2015; Crossley et al., 2017; Digumarti et al., 2014). It is also a convenient method to deal with the uncertainty in a given optimization.

3 | DESIGN OF A ROBOT

The primary purpose of designing a robot is to climb a telecom tower, inspect the electronic equipment installed at the top of the tower, and assist the rigger (Shah & Dave, 2023). The riggers climb the tower with a ladder attached to the tower. The proposed climbing robot uses the same existing ladders attached to the tower for climbing the tower. The types of ladders attached to towers and their dimensions are presented in Section 3.1. The dimensions of the proposed robot are highly dependent on the ladder dimensions. The design of the robot is done in solid works, and its mathematical calculation is done with the basic consideration of the size of the ladder, step height, distance to be travelled, inertia forces, weight of the robot, velocity and acceleration during the motion. Section 3.2 represents the design of all the components of the ladder-climbing robot. A detailed analysis of the robot is presented in Section 3.3.

3.1 | Ladders attached to telecom towers

There are two varieties of ladders widely used on telecom towers. One has the straight rungs and the other has zig-zag rungs. Figure 1 shows pictures of actual towers with ladders and their prototypes developed in the laboratory. Figure 2a shows the ground-based tower with a straight-rung ladder. Figure 2b shows the same type of ladder with exact dimensions, which is developed in the laboratory for testing and experiment. The zig-zag-type ladder is attached to the ground-based mast, as shown in Figure 2c. The prototype of the mast and ladder developed for experimentation is shown in Figure 2d.

The ladder dimensions are instrumental in identifying the proposed robot's dimensions. The dimensions of the robot's components are identified by considering the height between two rungs as a

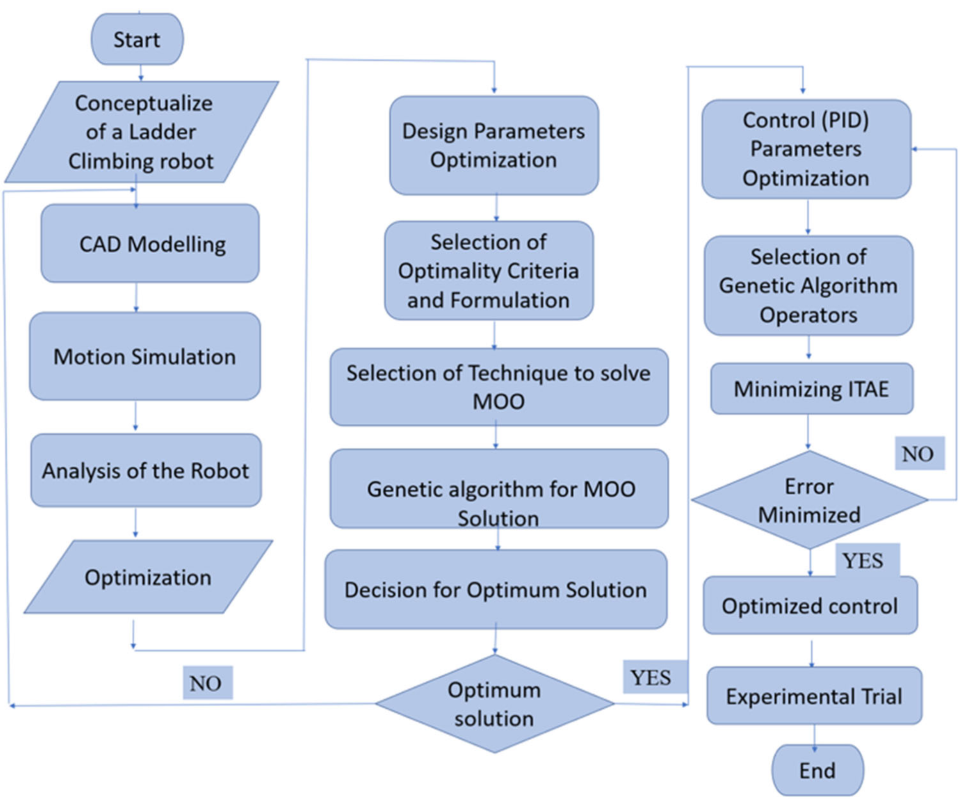

FIGURE 1 Design analysis and optimization methodology for a proposed robot. CAD, computer-aided design; ITAE, integral time absolute error; MOO, multiobjective optimization; PID, proportional–integral–derivative. [Color figure can be viewed at wileyonlinelibrary.com]

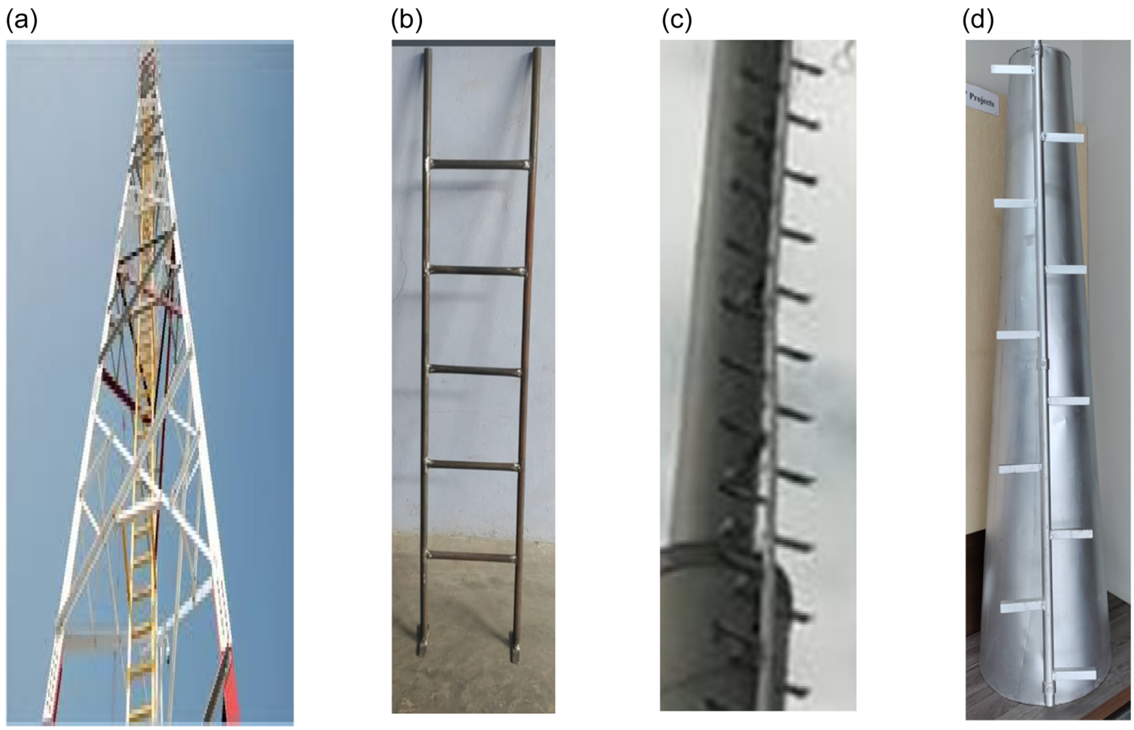

FIGURE 2 Types of the ladders on telecom towers for climbing. (a) The ladder on a telecom tower for climbing (Types, 2007), (b) a prototype of the ladder, (c) a ladder on the actual tower (Basics, Telecom Tower, 2017) and (d) a prototype of tower and ladder. [Color figure can be viewed at wileyonlinelibrary.com]

travel distance in a cycle. The OSHA's regulations for the standard ladder dimensions are shown in Figure 3 (OSHA Ladder Regulations, 2020).

3.2 | Design of a ladder-climbing robot

A robust, compact, simple ladder-climbing robot mechanism is designed considering workers' ease of operation and usage. The mechanism is designed using a double rack and pinion arrangement. It is pretty suitable to achieve linear vertical motion to climb ladders attached to the telecom tower. Four clamps-like gripping arrangements to advance climbing on rungs of a ladder ensure a firm grip on the ladder (Shah et al., 2024a). Motion transmission through mechanical arrangements offers fewer degrees of freedom than quadruped robots with similar motion. The proposed design is low-cost as fewer actuators are required due to lesser degrees of freedom, and it gives perfect climbing motion. Figure 4 shows the proposed robot mechanism and its movement on the ladder. Figure 5a–c shows a CAD model of the robot consisting of four clamps attached to a double rack and pinion arrangement and its motion simulations. Four clamps work as grippers and support robots at four places on the ladder rung. Figure 5d–f shows the workings of the robot prototype that can give satisfactory motion on the tower's ladder developed in the laboratory.

Due to the vertical motion required to climb the rung of a ladder, a double rack and pinion mechanism is adopted to design the robot

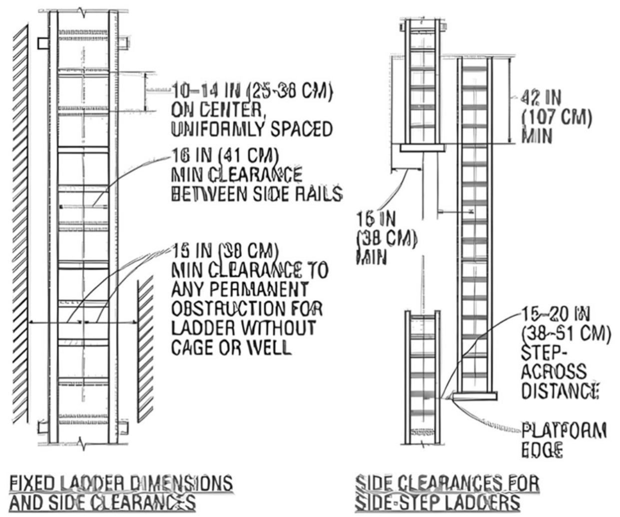

FIGURE 3 OSHA regulations for the ladder of the tower (OSHA Ladder Regulations, 2020). OSHA, Occupational Safety and Health Administration.

capable of climbing the ladder. The two-point contact gait pattern is adopted to climb the vertical ladder with almost 90° angles to the ground. Hence, static and dynamic stability analysis of a robot is essential.

The mathematical formulations of components like rack, pinion and clamps are achieved by considering inertia forces, reaction forces and the robot's weight. Certain assumptions while designing

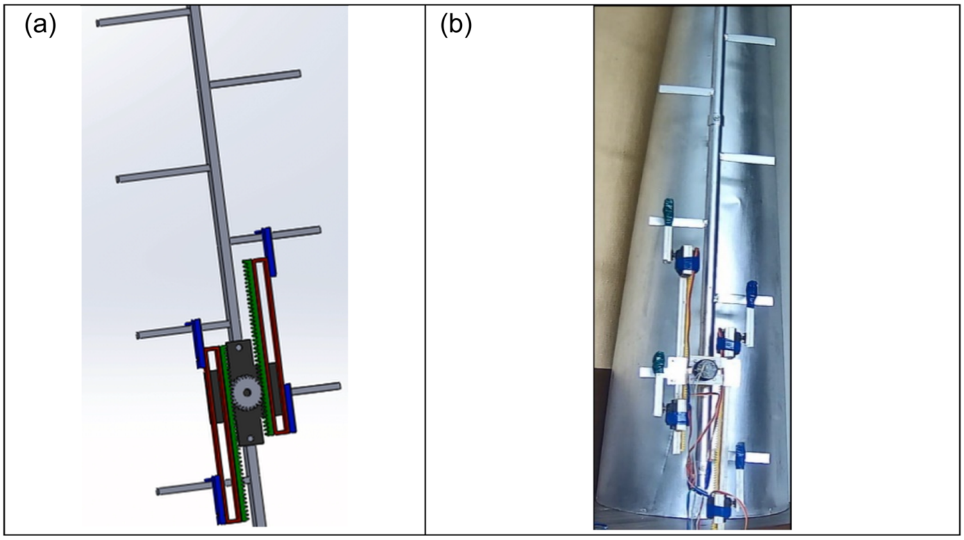

FIGURE 4 Proposed design and developed ladder-climbing robot. (a) Design of a ladder-climbing robot and (b) developed a ladder-climbing robot and a tower with a ladder. [Color figure can be viewed at wileyonlinelibrary.com]

are as follows. (1) Types and Dimensions of the ladders are taken as per OSHA's recommendations, as shown in Figure 3. (2) The payload on the robot is taken as 50 N, considering the weight of the inspection device. (3) The movement of the clamp to have the flexibility to adjust the 5-cm variation in rung dimensions. (4) The safety factor is considered 1.5 while designing components and selecting actuators.

3.2.1 | Design of pinion, rack and clamp

As shown in Figure 4a, the main motion-transmitting elements of the proposed robot are the pinion (input drive) and rack (output driven). The double rack and pinion design will help with motion on telecom tower ladders, as shown in Figure 5a–c. The pinion rotates alternately clockwise and anticlockwise to move the right and left rack upwards, allowing linear motion. Pinion, racks and clamps are designed, and critical dimensions are obtained. Rack length is taken by considering the distance between the ladder's rungs. Rack and pinion design parameters are shown in Table 1.

The double rack arrangement for the proposed robot actuates alternatively. When the pinion moves counterclockwise, the right rack moves up and archives a new upwards position on the rung of the ladder. Then pinion moves clockwise, and the left rack advances linear motion and archives the next step on the ladder. The pair of racks is designed as per the dimensions of the ladders having Module 3 and a pitch height of 10 mm. The robot is connected to ladder rungs with the help of a clamp, maintaining contact of the robot with the ladder. The clamps are designed based on the weight of the robot and the reaction of the ladder's rungs on each clamp. The dimensions of the clamps are taken concerning the distance between the two consecutive rungs of the ladder.

3.3 | Analysis of a climbing robot

The proposed ladder-climbing robot is analyzed to judge its motion characteristics. A systematic procedure is adopted for the analysis of the robot. The static stability analysis of robot is presented in Section 3.3.1. Considering the robot's self-weight and payload, basic forces and reactions are calculated. The dynamic stability analysis of a robot is represented in Section 3.3.2. As the ladder of towers is almost vertical, the dynamic stability of the robot is a crucial consideration.

3.3.1 | Static analysis of the climbing robot

Static stability of the robot is to be maintained through the weight and reaction forces on clamps. The two forces act on the robot when stationary on the ladder. The first force is the robot's weight, acting in a downward direction because of gravitational pull. The second force is the normal reaction from the ladder at the robot's clamps (R1, R2, R3 and R4), which is acting upwards. The total weight of the robot is 4.733 kg. Considering normal reaction (N) at a particular leg contact,

$$m * g = 4 * N, \tag{1}$$

$$\therefore \quad N = m * g/4 = 4.733 * 9.81/4 = 11.833\ N, \tag{2}$$

where m is the mass of the robot in kg, N is the normal reaction and g is the gravity constant. To have static stability for the robot, all the reactions applied by the ladder on the robot at all contact surfaces must be greater than N. Hence,

$$R1, R2, R3, R4 \geq N. \tag{3}$$

The necessary condition to be fulfilled to maintain the static stability of the robot on the ladder is given in Equation (3).

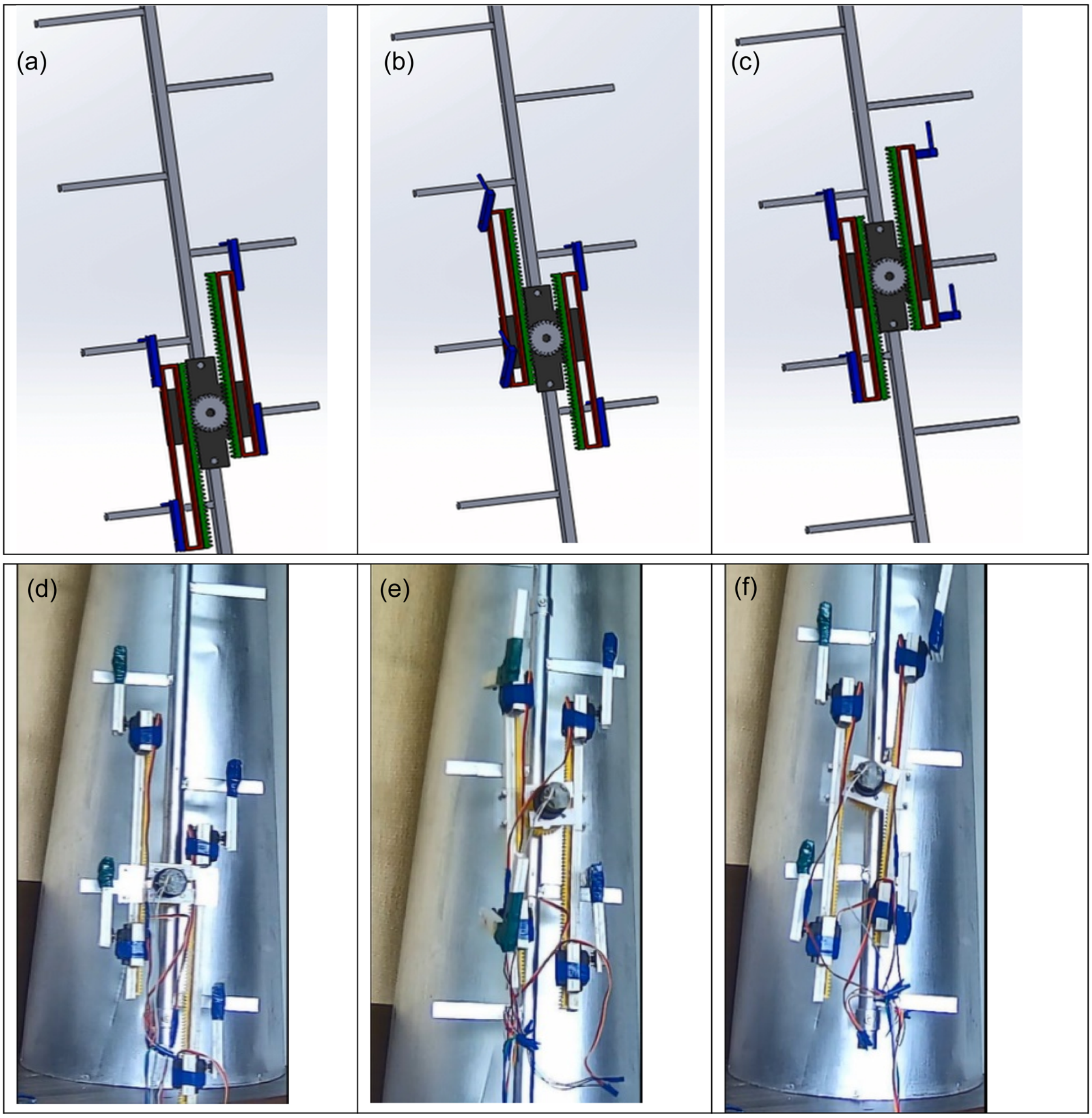

FIGURE 5 Computer-aided design model simulation and motion of the working prototype. (a) Initial position, (b) left clamp opens, (c) right rack advances to climb, (d) initial position, (e) left clamp opens and (f) right rack advances to climb. [Color figure can be viewed at wileyonlinelibrary.com]

3.3.2 | Dynamic stability analysis of the climbing robot

Dynamic analysis of a robot is important to have the robot's stability while in motion on the ladders of telecom towers. This section contains mathematical modelling of the system to identify dynamic forces at each component. The approach adopted here is to identify the second-order differential equations of motion for all the motion variables. The derived equations are solved using Laplace transformation, and transfer functions are developed. The developed transfer functions are plotted in MATLAB Simulink to determine stability and identify the controlling parameters.

The robot's motion is identical on both sides. Its dynamic analysis is performed for half a section of the robot. As the robot's design is symmetric, a 3D model of the half mechanism of the robot is shown in Figure 6.

The driver motor is attached to the pinion, resulting in the rack's linear motion. Figure 7a shows the pinion's free-body diagram.

Considering the inertia torque at the pinion, the equation of motion is given as

$$I_1\ddot{\Theta} = T_1 - RF_g, \tag{4}$$

where I_1 is the total inertia at pinion ($I_1 = I_p + I_m$), I_p the pinion inertia, I_m the motor inertia, $\ddot{\Theta}$ the angular acceleration of the pinion, T_1 the input motor torque, R the radius of pinion and F_g the tangential force.

The displacement at the clamp or gripper is taken as y. The rack displacement is x ($x = R\Theta$). Free-body diagrams of a

TABLE 1 Rack and pinion design parameters.

Sr. No.	Description	Remark
1	Material	Mild steel
2	Ultimate tensile strength S_{ut}	350 N/mm^2
3	Power	0.125 kW
4	Pressure angle α	20°
5	Lewis's form factor value y	0.364
6	Number of teeth (pinion)	32
7	Module	3
8	Width	10 mm
9	Rack length	300 mm
10	Clamp dimensions	124 × 124 × 24 mm L shape

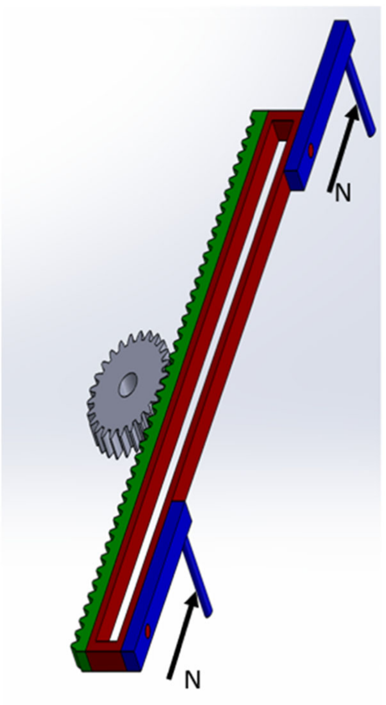

FIGURE 6 Modelling of rack and pinion. [Color figure can be viewed at wileyonlinelibrary.com]

rigid body combining pinion, rack and gripper are shown in Figure 7b,c.

Taking inertia force, restoring force and damping force for the rack,

$$(M_1\ddot{x}) = F_g + F_{s1} + F_{d1}, \tag{5}$$

where F_{s1} and F_{d1} are the restoring and damping forces, respectively, and M_1 is the mass of the rack. Taking displacements, stiffness and damping into account for the restoring and damping force,

$$(M_1\ddot{x}) = F_g + c(\dot{y} - \dot{x}) + k(y - x),$$

and further simplifying it,

$$(M_1\ddot{x}) + (c\dot{x}) + (kx) = (ky) + (c\dot{y}) + F_g. \tag{6}$$

Taking, M_2 is the mass of the gripper or clamp (Figure 7c), the dynamic equation is

$$(M_2\ddot{y}) + (c\dot{y}) + (ky) = (kx) + (c\dot{x}). \tag{7}$$

For the rack substituting displacement $x = R * \Theta$, substituting, linear velocity $\dot{x} = R * \dot{\theta}$ and linear acceleration $\ddot{x} = R * \ddot{\theta}$ in Equation (6),

$$(M_1R\ddot{\theta}) + (cR\dot{\theta}) + (kR\Theta) - (ky) - (c\dot{y}) = F_g. \tag{8}$$

Substitute F_g from Equation (8) into Equation (4),

$$I_1\ddot{\theta} = T_1 - R[(M_1R\ddot{\theta}) + (cR\dot{\theta}) + (kR\Theta) - (ky) - (c\dot{y})]. \tag{9}$$

Simplifying Equation (9),

$$[I_1 + (M_1R^2)]\ddot{\theta} + (cR^2)\dot{\theta} + (kR^2)\Theta = T_1 + (kRy) + (cR\dot{y}). \tag{10}$$

Defining
$I_e = I_1 + (M * R^2)$, $C_e = C * R^2$ and $K_e = K * R^2$ in Equation (8),

$$(I_e\ddot{\theta}) + (C_e\dot{\theta}) + (K_e\Theta) = T_1 + (kRy) + (cR\dot{y}). \tag{11}$$

Now using Laplace transformation for Equations (7) and (11),

$$(M_2s^2 + cs + k)Y(s) = (kR + cRs)\Theta(s) \tag{12}$$

and

$$(I_es^2 + C_es + K_e)\Theta(s) = T_1(s) + (cRs + kR)Y(s). \tag{13}$$

From Equation (12),

$$\Theta(s) = \frac{(M_2s^2 + Cs + K) * Y(s)}{(kR + cRs)}. \tag{14}$$

Putting into Equation (13),

$$(I_es^2 + C_es + K_e)[(M_2s^2 + c_s + k)Y(s)] = T_1(s)[kR + cRs] + [kR + cRs]^2Y(s). \tag{15}$$

Simplifying Equation (15),

$$Y(s) = \frac{kR + cRs}{[(I_eM_2s^4) + (I_eCs^3) + (I_eKs^2) + (C_eM_2s^3) + (K_eM_2s^2)]}T_1(s),$$

which gives

$$T_{f1} = \frac{Y(s)}{T_1(s)} = \frac{kR + cRs}{[(I_eM_2s^4) + (I_eCs^3) + (I_eKs^2) + (C_eM_2s^3) + (K_eM_2s^2)]}, \tag{16}$$

where T_{f1} represents the transfer function for rack and pinion.

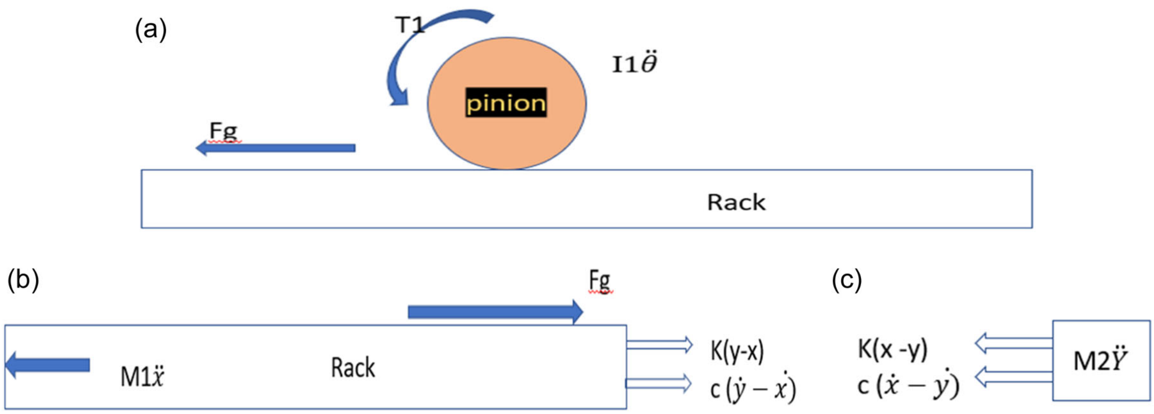

FIGURE 7 Free-body diagrams of components. (a) Free-body diagram of pinion and rack, (b) free-body diagram of a rack and (c) free-body diagram of a gripper. [Color figure can be viewed at wileyonlinelibrary.com]

Derivation for transfer function for clamp attached to rack with a servo motor torque T_2 is given as

$$(I_2\ddot{\theta}_1) + (ct\dot{\theta}_1) + (kt\theta_1) = T_2. \tag{17}$$

Applying Laplace transformation to Equation (14), $(I_2 s^2 + c_t s + kt)\Theta(s) = T_2(s)$ and transfer function

$$T_{f2} = \frac{\Theta(s)}{T_2(s)} = \frac{1}{(I_2 s^2 + c_t s + kt)}. \tag{18}$$

Equations (16) and (18) show the transfer function of the half part of the robot. The system parameter in the equation represents the mass, damping and stiffness properties. The CAD model helps us to give mass properties and the total mass of the robot. The mass properties help derive damping and stiffness properties. The final transfer functions arrived using the mass properties from the CAD model as

$$T_{f1} = \frac{0.25s + 0.5}{0.125s^4 + 2s^3 + 1.85s^2 + 20} \quad \text{and} \qquad T_{f2} = \frac{1}{0.01s^2 + 2s + 1}. \tag{19}$$

The system's derived transfer functions represent the robot's dynamic behaviour. These are simulated using MATLAB Simulink to obtain critical values of controlling parameters.

4 | MOO OF ROBOT

On the basis of a critical examination of the desired automation system's attributes like motion parameters, size of components and performance requirements of the ladder-climbing robot, mathematical formulation is done to establish the objective function and constraint equations for optimization. Section 4.1 highlights the formulation of the MOO problem. Section 4.2 includes in-depth information about the solution technique used, and Section 4.3 contains the solution to the MOO problem. Section 4.4 consists of the optimization of the robot's controlling parameters.

4.1 | Formulation of the objective functions

For successfully climbing a robot on a telecom tower for inspection and maintenance, selecting optimality criteria and formulating the objective function is very important (Crossley et al., 2017). Optimizing the minimum weight and natural frequency of vibration is required. Minimizing weight and natural frequency are two critical optimization criteria that are contradictory in nature. It is important to discuss each criterion and the interconnection between them.

Reducing the weight of the climbing robot is crucial for several reasons, such as improved efficiency, enhanced portability, lower cost and reduced load on the telecom tower's ladders. A lighter robot requires less climbing energy, resulting in longer battery life and increased operating time. Lighter materials and components can lead to reduced manufacturing costs. The lighter robot is easier to transport and deploy on different telecom towers. Minimizing the robot's weight lessens the load on the ladder rungs, decreasing wear and tear on the ladder itself.

Optimizing the natural frequency ensures stability, safety and performance during ladder climbing and operation. A robot with an appropriately tuned natural frequency is less likely to experience resonance or instability during ladder climbing, improving its overall stability and reducing the risk of accidents. A robot with a well-controlled natural frequency can dampen vibrations caused by ladder interaction or external disturbances, leading to better precision and reduced wear on its components. Designing for a specific natural frequency allows engineers to create a robot with a sturdy structure that can withstand ladder-induced forces and maintain its performance over time.

Both criteria are contradictory and interconnected. The weight of a robot affects its natural frequency and vice versa. A lighter robot tends to have a higher natural frequency because it typically has lower inertia and can respond more rapidly to external forces. Conversely, a heavier robot may have a lower natural frequency, reducing stability and increased vibration effects. Achieving a balance between weight and natural frequency is crucial. While a lighter robot is generally desirable for energy efficiency and portability, it should not compromise the robot's structural integrity and stability. A well-optimized design minimizes weight without sacrificing the required natural frequency.

An ideal design achieves a lightweight structure with a well-tuned natural frequency, ensuring stability, safety, efficiency and overall performance during telecom tower maintenance tasks. The formulation of the objective functions for optimization is done precisely with mathematical techniques. The objective function 1 for the minimization of weight is

$$f_1(x) = 1110\left[\left(\frac{\pi}{4}*x_4{}^2*x_1\right) + 2(x_1*x_2*x_3) + 4\left(0.005*\frac{x_3}{3}\right)\right] \quad (20)$$

and the objective function 2 for minimization of natural frequency is

$$f_2(x) = \sqrt{\frac{0.03*x_2*0.35*10^{11}}{1.88*x_1 + 0.03*x_2*x_3}}. \quad (21)$$

The design variables are the dimensions of the robot and constraints on them are

$$\left.\begin{array}{l} g_1(x) = 0.015 \le x_1 \le 0.04, \\ g_2(x) = 0.009 \le x_2 \le 0.015, \\ g_3(x) = 0.3 \le x_3 \le 0.4, \\ g_4(x) = 0.04 \le x_4 \le 0.06. \end{array}\right\} \quad (22)$$

The variable x_1 is the rack's thickness, x_2 the pinion and clamp's width, x_3 the length of the rack and x_4 the diameter of the pinion. On the basis of the initial conceptual design, the design vector is [0.02 0.015 0.3 0.054]. The clamp length is taken as 1/3 of the rack length, and the thickness of the clamp is 5 mm. The material of the robot is considered as plastic for analysis.

4.2 | Techniques for MOO solution

There are many other methods to solve such multicriteria problems. Traditional techniques Gradient Descent and Linear Programming have limitations to deal with the nonlinearity of the robot optimization problems. Other involutory algorithms like PSO might get struck in local minima and differential evolution may need careful parameters tunning. Recent advancements in Reinforcement Learning have demonstrated its potential in robotic control. However, the high computational cost and the need for extensive training environments can be a drawback compared with GAs. The proposed GA method for the design and MOO of a two-point contact ladder-climbing robot lies in its robustness, flexibility and ability to handle complex, nonlinear and multimodal optimization problems. Its global search capability, adaptability to multiple objectives and practical ease of implementation make it an excellent choice for this application compared with traditional optimization methods, other evolutionary algorithms and machine learning-based approaches. In most traditional methods, the solution towards the convergence depends on the initial guess values for the chosen solution. Due to guess value, it may tend to struck a suboptimal solution. For most of the multiple objective problems with conflicting objectives, like, variation in one objective adversely affects the other objective, a GA is the best suitable technique. It achieves a faster rate towards convergence. Section 4.2.1 highlights the capabilities and suitability of the GA for the formulated problem. The procedure adopted to solve the problem using a GA is presented in Section 4.2.2.

4.2.1 | Genetic algorithm

The GA approach possesses many advantages over the other techniques. The classical processes are time-consuming, tedious and expensive. Adopting a biologically inspired evolution algorithm helps solve problems faster and discovers the best specific solution from many available solutions. The GA suits most of the problems as it is metaheuristic in nature.

There are two basic approaches to solving multiobjective genetic algorithm (MOGA) problems. The multiobjective GAs are broadly categorized into two categories, namely, Pareto- and decomposition-based MOGAs. The decomposition method decomposes the multiple objectives problems into subproblems of a single objective, and each subproblem's solution is obtained concurrently. It is termed a multiobjective evolutionary algorithm decomposition method. It is a primary method and needs more time to reach convergence than the Pareto front optimization technique (Bu et al., 2022; Deb, 1995; Kouritem et al., 2022; Petrović et al., 2022). The main goals of MOGA problems are coverage, convergence and diversity. The Pareto front is a set of all efficient solutions. It is a widely used concept focusing on the most dominant solution results sets. The highly efficient solution set points are the Pareto front obtained using a nondominant sorting GA. Researchers developed techniques called NSGA, NSGA-II and NSGA-III (Deb & Jain, 2014; Srinivas & Deb, 1994). The distinction between all three is discussed in Table 2.

TABLE 2 Difference between NSGA, NSGA-II and NSGA-III.

NSGA	NSGA-II	NSGA-III
– NSGA is a popular nondomination-based genetic algorithm for multiobjective optimization. – It is a very effective algorithm but has been generally criticized for its computational complexity, lack of elitism and for choosing the optimal parameter value for sharing parameter share.	– It uses an elitist principle; the elites of a population are given the opportunity to be carried to the next generation. – It uses an explicit diversity-preserving mechanism (crowding distance). – It emphasizes the nondominated solutions.	– Extended version of NSGA-II to deal with a many-objective optimization problem, using a reference point approach, with a nondominated sorting mechanism. The newly developed algorithm is called NSGA-III.

Abbreviation: NSGA, nondominated sorting genetic algorithm.

The NSGA-II is a well-known, fast sorting and elite MOGA among all the three algorithms. NSGA-II simultaneously optimizes each objective without being dominated by any other solution. In the present multiobjective problem, the NSGA-II algorithm is adopted and Pareto front solution sets are obtained.

4.2.2 | Procedure adopted to solve the problems

The authors propose to use MOGAs to optimize component diameter, link lengths and other design parameters to determine the design of the robot for the required fitness function (Bhoskar et al., 2015; Bjorlykhaug & Egeland, 2018; Datta et al., 2016; Konak et al., 2006; Zeid et al., 2024). The objective functions are formulated considering minimizing the weight and natural frequency of the system. GA is inspired by biological evolution and involves the following stepwise procedure:

Step 1: The first step is to select the population and size for the defined objective function to fit the genetic representation. The formulated objective functions are the fitness functions to be optimized. A tournament selection for the population is adopted.

Step 2: The initial population size is considered based on the problem and criteria to be optimized. The initial population generation is random, and the population size is taken as 100.

Step 3 selection: A portion of the old existing population is allowed to breed a new successive generation. The individuals are selected based on their fitness and features. Through a fitness-based process, individual solutions are chosen. The quality of the solution depends majorly on the selection process.

Step 4 genetic operators: The population selection of the generation must include elite individuals. A combination of mutation and crossover is included in the selection of parents for the production of the next generation. Generated child solutions using crossover and mutation methods typically share their parents' features. The process continues for the next generation, and the selection of parents for the new generation is to be done up to the appropriate population size. The chromosomes of the next generations are different from the initial generation. This stage is very important for elite features in the generations to be included. A two-point crossover with a 0.8 level is taken as a lower rate may lead to slower convergence. The mutation rate is taken as 0.05 as a higher rate may degreed the quality of the solution.

Selection of parameters GA is based on the rigorous analysis of the effects of each parameter on the results of optimization. The choice of values for genetic operators, like, population size, crossover rate and mutation rate, is critical in determining the performance and behaviour of a GA. While optimizing, the genetic operator rates are selected to arrive at the solution. Population Size should be large enough to maintain diversity but not excessively large to avoid unnecessary computational burden. Population size is chosen based on the number of variables and the complexity of the problem. The optimal crossover rate often depends on the problem domain, with some problems benefitting from higher crossover rates while others perform better with lower rates. Mutation helps maintain genetic diversity and prevents stagnation. The optimal mutation rate depends on the problem's characteristics and the balance between exploration and exploitation desired by the algorithm. Their sensitivity with objective functions and PID parameters is checked to select these parameters.

The sensitivity of the objective functions and PID parameters optimization are checked using the Monte-Carlo simulation technique for different population size values, crossover rates and mutation rate values. The number of combinations of these parameters is checked for both the objective function and, based on the results, the parameter values are chosen. The results of one of the objective functions showing sensitivity with all three parameters are included herewith just for reference. Figure 8 represents the analysis results of the objective function of the weight of the robot.

The effect of population size, crossover rate and mutation rate is shown In Figure 8a–c. Looking at less computation time, fast convergence and elite features inclusion (Hassanat et al., 2019), the selected values of population size = 100, crossover rate = 0.8 and mutation rate = 0.05.

Step 5 heuristics: The heuristics operators must be employed to arrive at robust results faster.

Step 6 termination: The termination criteria must be decided initially before starting the problem's solution. The conditions to be considered are satisfying minimum criteria, the final number of iterations reached, the budget allocated for computation, the highest fitness achieved, manual intervention or a combination of the above. The termination criteria is taken as the maximum number of iterations reached.

Constraints are the lower and upper bounds of the values of design variables. Handling constraints in the presented optimization process for a two-point contact ladder-climbing robot involves evaluating solution based on Pareto front that considers both objectives and explicit constraints. To obtain a solution, NSGA-II is adopted, and trade-off analysis is performed to balance conflicting objectives. A careful selection of mutation and crossover strategies, maintains diversity and the optimization process can effectively navigate the search space to find feasible and optimal solutions that meet all required constraints.

4.3 | Solution of MOO with GA

The objective functions shown in Equations (20) and (21) and constraints in Equation (22) are solved in MATLAB, and the resulting Pareto front is obtained, as shown in Figure 9. The parameters considered while solving the problem are indicated in Table 3.

The Pareto front obtained shows the nondominated shorted points representing the problem's solution range. Towards the extreme left top corner, objective function 1 is optimum, and in the extreme right bottom corner, objective function 2 shows the most desirable value. That means most optimum results in one objective are obtained at the cost of the other objective. The MOO problem is solved by trading off between the two goals. Considering the knee decision approach, the optimum point is decided, and optimized parameter values are obtained. The minimum weight of the robot, which is made of plastic material, is 2.6 kg, and the natural frequency of the robot is 12.26 rad/s. The optimized design vector is [0.021, 0.012, 0.32, 0.05]. Appendix 1 indicates the results GA code. The optimized objective function values are highly sensitive to the variables' value. To obtain the sensitivity of variables in optimization, sensitivity analysis is performed.

The authors use Pareto-based sensitivity analysis for sensitivity analysis of a MOO problem with four variables using a GA in MATLAB. This method involves analyzing the Pareto front generated by the GA to understand the sensitivity of the objectives to changes

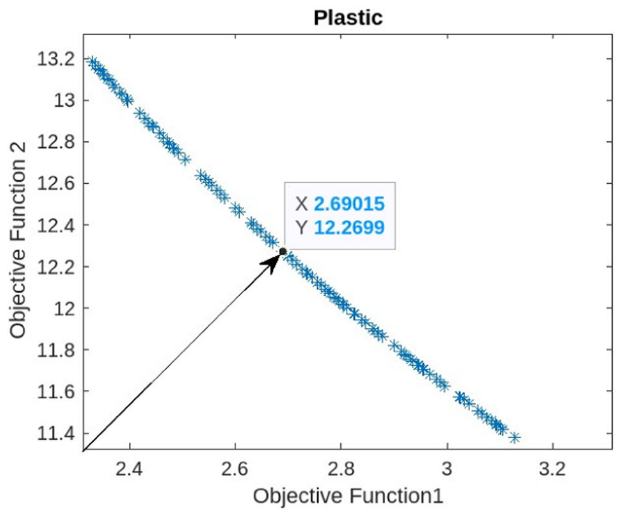

FIGURE 9 Pareto front solution of multiobjective optimization solution. [Color figure can be viewed at wileyonlinelibrary.com]

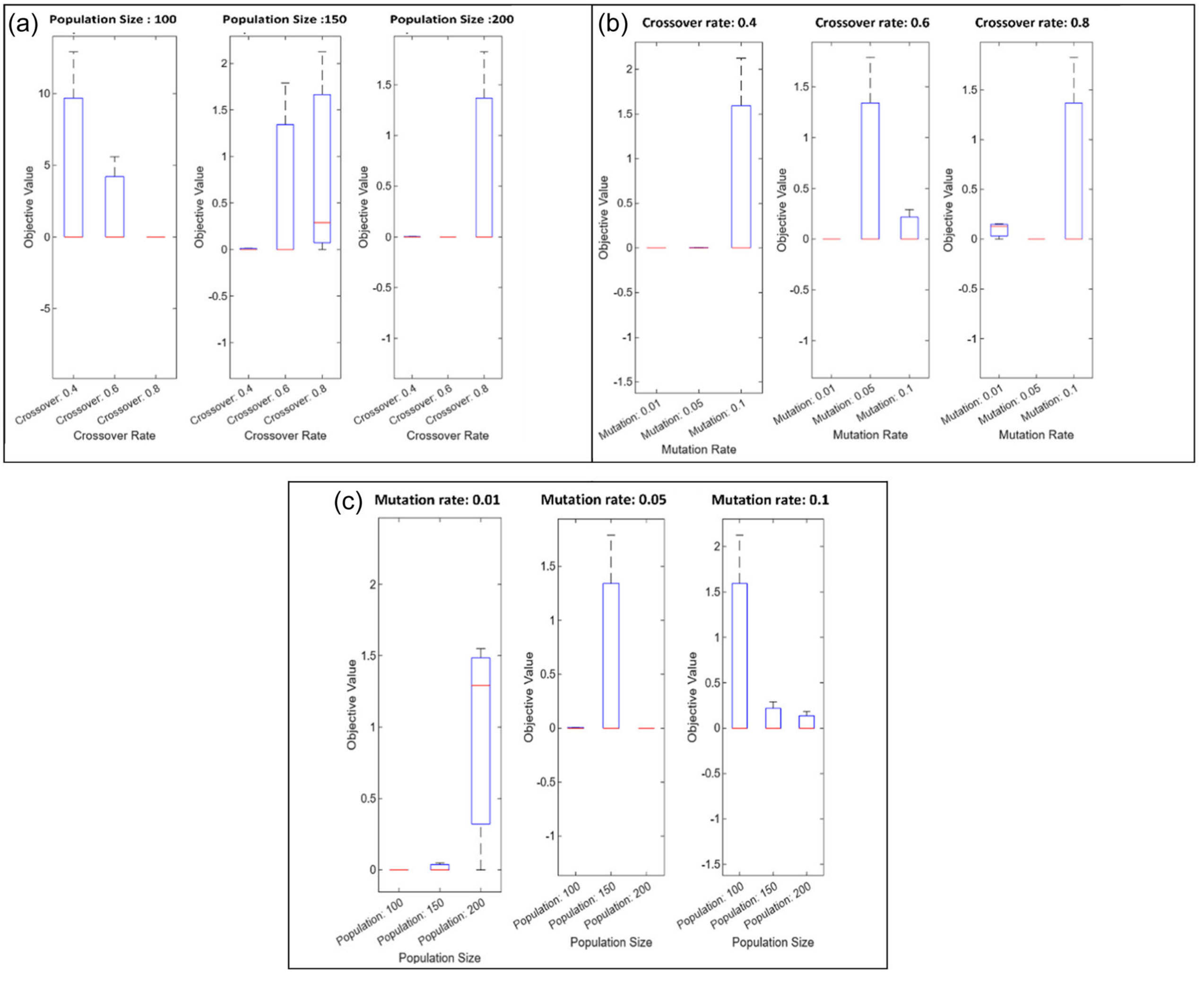

FIGURE 8 Sensitivity of genetic operators on the objective function. (a) Sensitivity of crossover rate with population size, (b) sensitivity of mutation rate with crossover and (c) sensitivity of population size with mutation. [Color figure can be viewed at wileyonlinelibrary.com]

in the input variables. The sensitivity analysis code is developed in MATLAB 2022, and its results are obtained. Figure 10a–d shows the effects of variations in the robot dimensions on the robot's weight. Figure 11a–d shows the variations in dimensions with the robot's natural frequency.

TABLE 3 Genetic algorithm parameters for multiobjective optimization solution.

Description	Selection of parameter
Fitness function	Objective functions
Number of variables	4 $[x_1, x_2, x_3, x_4]$
Lower bounds	[0.015 0.009 0.3 0.04]
Upper bounds	[0.04 0.015 0.4 0.06]
Population size	100
Selection criteria	Uniform
Fitness scaling function	Rank
Selection type	Tournament
Reproduction—crossover function	0.8 (two points)
Mutation	0.05

The relations shown in Figure 10 show that the variation in the robot's dimensions is proportional to the mass. A Lesser mass is desired, but it should not compromise strength or other parameters. Figure 11a–d shows the variations in dimensions with the robot's natural frequency. The rack thickness and pinion diameters are inversely proportional to the natural frequency. These indicate the effects of variation in all the variables and their impact on the values of objective functions. Once the optimization is performed and the optimal values of the parameters are obtained, the sensitivity analysis is conducted to understand the impact of variations in these parameters on the robot's performance. The results obtained from this analysis are to decide the dimensions of the robot.

4.4 | GA-based PID parameters optimization

A GA is a technique to solve optimization problems based on biological evolution. The method is used to solve any optimization problems and provide robust and better solutions than other methods. The GA approach to solving the PID parameters optimization problem is discussed in Section 4.4.1. Optimizing PID controller parameters is essential to achieve the desired smooth, jerk-free

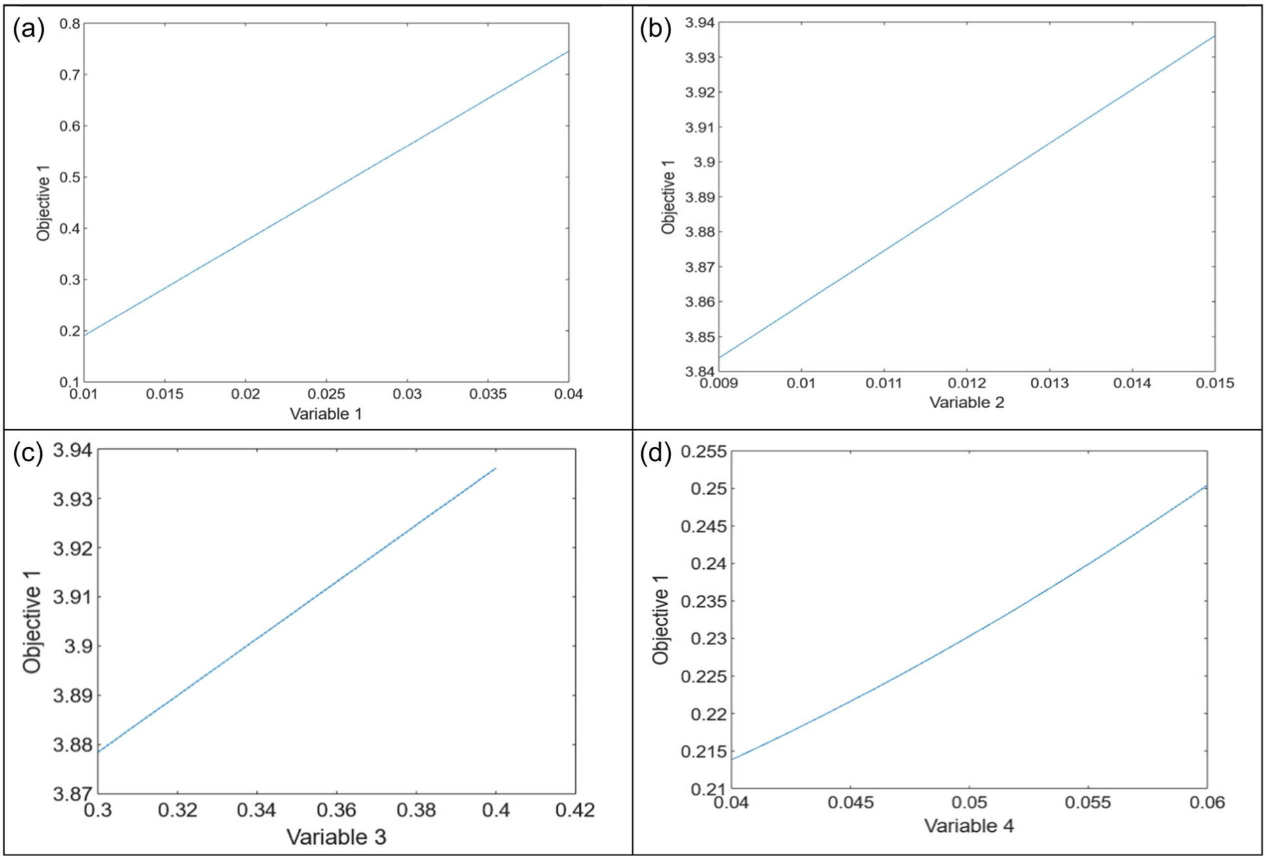

FIGURE 10 Variations in the mass of the robot with its dimensions. (a) Variation in the mass of the robot with the rack thickness, (b) variation in the mass of the robot with the pinion width, (c) variation in the mass of the robot with the rack length and (d) variation in the mass of the robot with the pinion diameter. [Color figure can be viewed at wileyonlinelibrary.com]

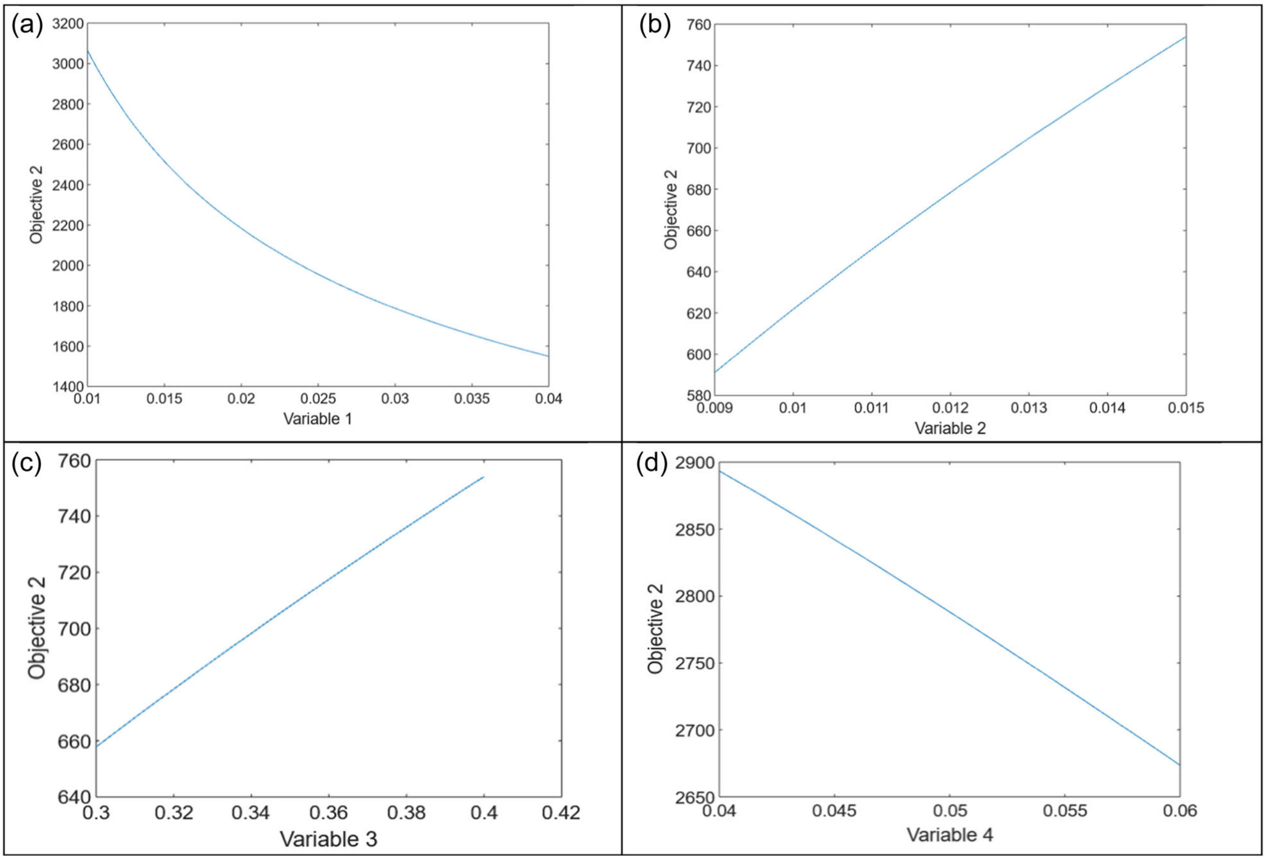

FIGURE 11 Variations in the natural frequency of the robot with its dimensions. (a) Variation in the natural frequency with rack thickness, (b) variation in the natural frequency with pinion width, (c) variation in the natural frequency with rack length and (d) variation in the natural frequency with pinion diameter. [Color figure can be viewed at wileyonlinelibrary.com]

motion. To avoid motion problems of a robot climbing a ladder with stability, fine-tuning of the PID controller parameters Integral Gain (k_i), proportional gain (k_p) and derivative gain (k_d) is required. Transfer functions from the dynamic modelling of the ladder-climbing robot are implemented using MATLAB Simulink. The Simulink model of the system with PID controller and GA integration is elaborated in Section 4.4.2.

4.4.1 | GA approach

The solution to optimize PID parameters is done using the GA technique. It utilizes a biologically inspired evolutionary algorithm based on the natural selection process. In this metaheuristic technique, a candidate solution to a problem is evolved towards better results via stagewise iterative progress. Figure 12 shows the flow chart for the methodology to solve the problem with the GA. Initial population selection forms the upper and lower bounds of the associated problems. The optimization process starts with the randomly selected population of individuals, and then the evolution of the next-generation population occurs via successive iterations. Reproduction, crossover and mutation are the stages required to be followed in each set of iterations to achieve better results. The fitness function is the objective function to be checked for every individual in the population. The fittest individual is selected stochastically from the population; its genome characteristics are modified to generate next-generation populations. The population generated is now used for the next iteration. The algorithm's termination condition is to finish a maximum number of iterations, and the fitness function value is obtained for the population.

4.4.2 | Optimization and system model integration

Identify a combination of the PID controller parameters such that the robot's resulting motion is stable and jerk-free while climbing the ladder. The transfer functions of the system that arrived from the dynamic analysis in Section 3.3 are simulated in Simulink. The system response plot is achieved using the PID controller. The response shows a very high overshoot with a periodic response. This indicates that it is required to choose values of an Integral gain (k_i), proportional gain (k_p) and derivative gain (k_d) to arrive at an aperiodic (without overshoot) response to the system. The fitness function for the optimization is considered an integral time of the absolute error, as shown in Equation (23).

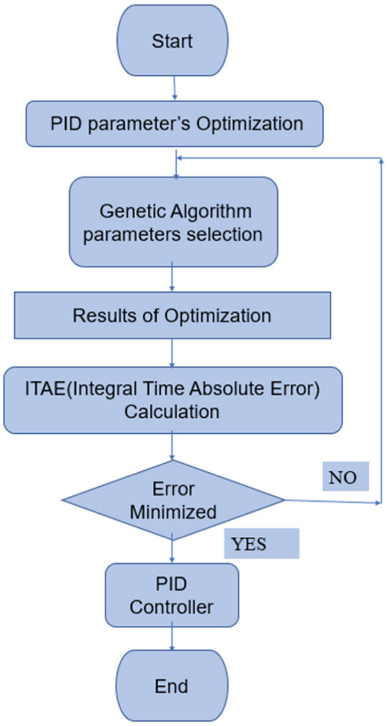

FIGURE 12 Methodology to apply the genetic algorithm for PID parameters. PID, proportional–integral–derivative. [Color figure can be viewed at wileyonlinelibrary.com]

$$\mathrm{ITAE} = \int_{t=0}^{t=\mathrm{final}} |\triangle f| * t\, dt. \tag{23}$$

The GA optimization is performed in the MATLAB Simulink in coordination optimization toolbox version 2016. Table 4 highlights parameters considered for simulation trials for optimizations.

The obtained optimized values of parameters are required to be inserted in the PID block of MATLAB Simulink, as shown in Figure 13. Figure 13 shows the schematic diagram of the system model integrated with ITAE. The bottom part of the diagram indicates the system's performance without optimization. The red line in Figure 14 indicates the system response with overshoot. Optimized PID controller parameters k_p, k_i and k_d result in smooth aperiodic motion. The blue line in Figure 13 shows the result with optimized parameters.

Simulation for optimization of PID parameters is done in MATLAB Simulink. It took 51 iterations to arrive at the final results. Appendix 2 shows the results of all iterations in MATLAB. GA plots for best fitness, fitness for each individual, Number of Children in each generation and entire generations are shown in Figure 15.

5 | DEVELOPMENT OF A ROBOT PROTOTYPE

Mechanical components like racks, gears, pinions and clamps are procured, and a prototype of the robot is developed in the laboratory. Electrical and electronic components used in the robot are a DC motor with an encoder, four servo motors, a 12-V battery, electrical wires, an Arduino microcontroller, and a camera for inspection purposes. The DC motor with a 12-kg-cm holding torque capacity is used as the driving motor of the pinion. It works on a power supply of 12 V, making electrical connections from the DC motor to the battery. The microcontroller used is Arduino UNO that controls the motion of the robot. Electrical connections are made from the DC motor to Arduino. The DC motor requires 12 V, and Arduino can supply only 5 V; an external 12-V DC battery is required to supply power to the DC motor. Four servo motors are used for the motion of four legs. Clamps are assembled to the motor. The microcontroller used is Arduino UNO. The camera is ESP32-CAM 2 MP with Bluetooth, WiFi and the Development Board. The camera is connected to Arduino via jumper cables. A camera is used for inspecting and taking pictures at the height. The electrical circuit diagram for all connections and camera attachments in the simulation environment is shown in Figure 16.

TABLE 4 Genetic algorithm parameters.

Description	Selection of parameter
Fitness function	ITAE
Number of variables	3 (k_p, k_i and k_d)
Lower bounds	[0 0 0]
Upper bounds	[100 100 100]
Population size	50
Selection criteria	Uniform
Fitness scaling function	Rank
Selection type	Tournament
Reproduction—crossover function	0.8 (two points)
Mutation	0.05

Abbreviation: ITAE, integral time absolute error.

As shown in Figure 4, a robot prototype is developed with plastic material, and a motion algorithm is developed, which gives a satisfactory climbing motion of the prototype on the tower's ladder. Figure 17 shows the results of the experimental trial of a climbing robot. Figure 18 shows the pictures taken with the help of a camera attached to the robot. It indicates the presented robot performs satisfactory climbing motion and can take pictures on the top of the tower and perform the inspection task.

The design of the robot for climbing ladders of telecom towers is designed and its static and dynamic analysis is performed. Using the developed mathematical formulation, the design parameters and controller parameters of the robot are optimized. The robot is then developed with the optimized parameters and tested considering the optimized control parameters. The trials of the developed robot are taken, and smooth climbing motion is achieved. That validates that the adopted

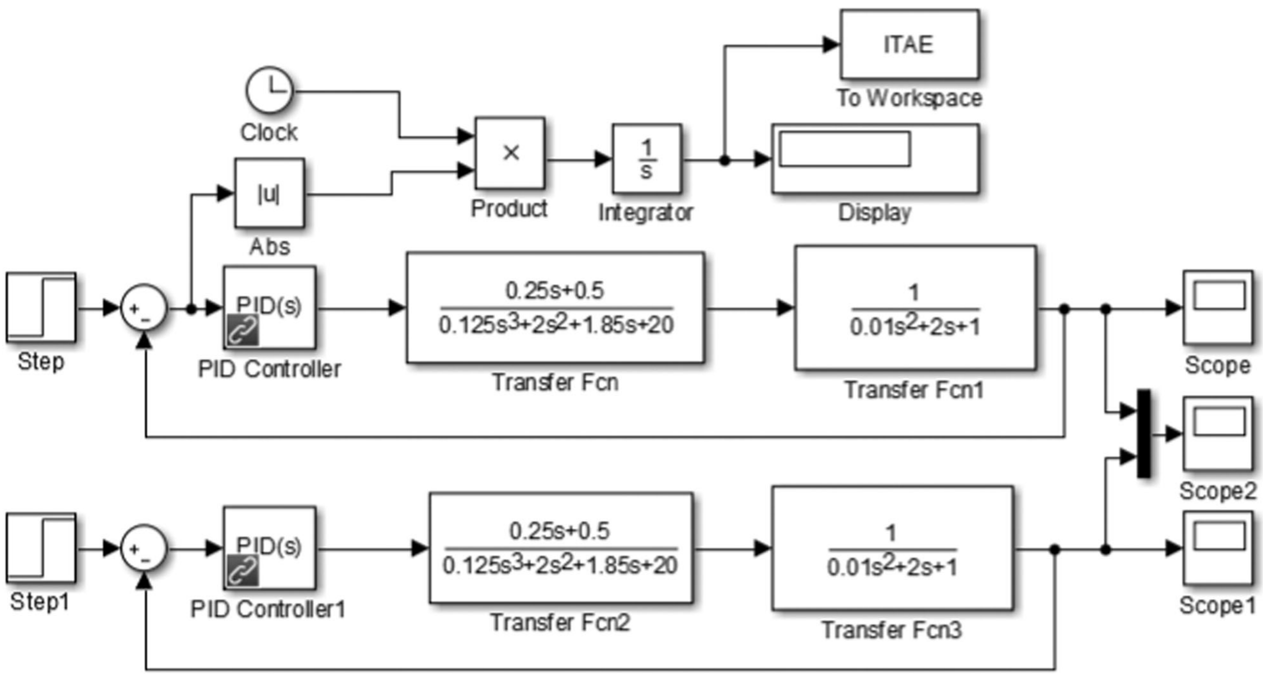

FIGURE 13 Simulink model with ITAE integration. ITAE, integral time absolute error; PID, proportional–integral–derivative.

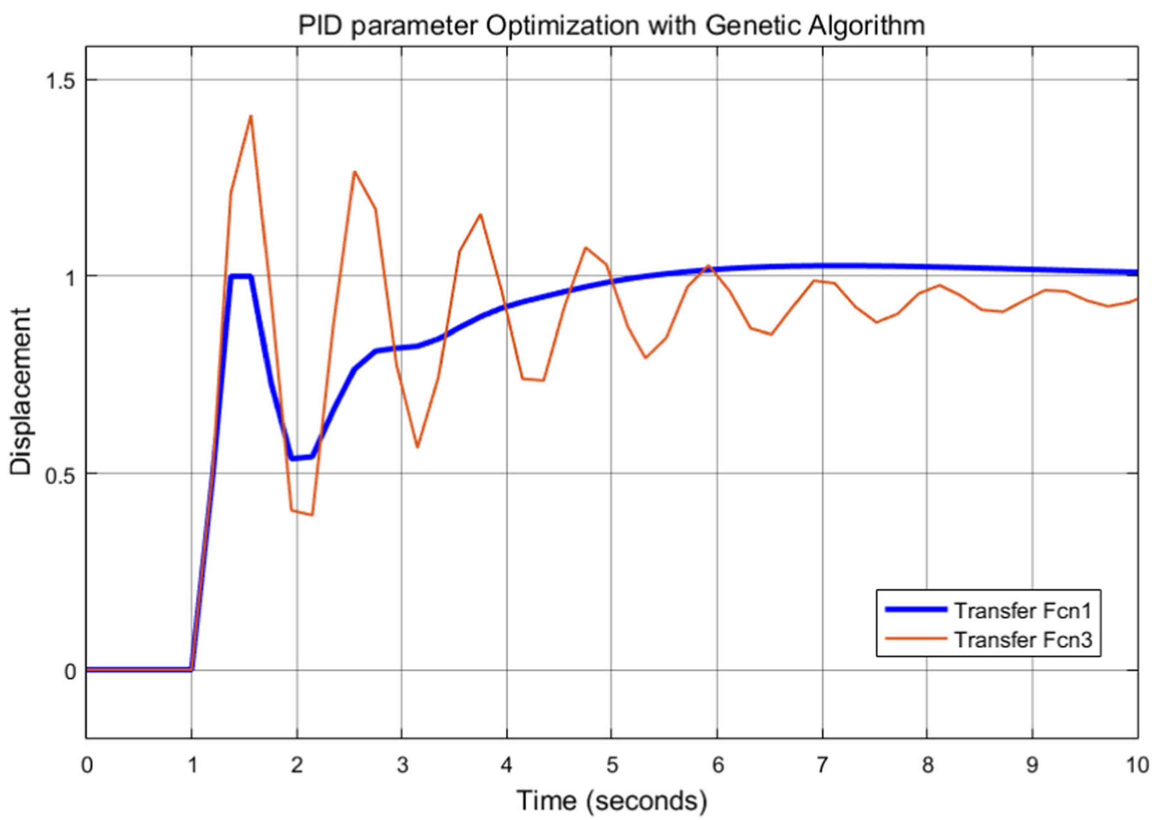

FIGURE 14 Comparison of results with and without optimized PID parameters (k_p, k_d and k_i). PID, proportional–integral–derivative. [Color figure can be viewed at wileyonlinelibrary.com]

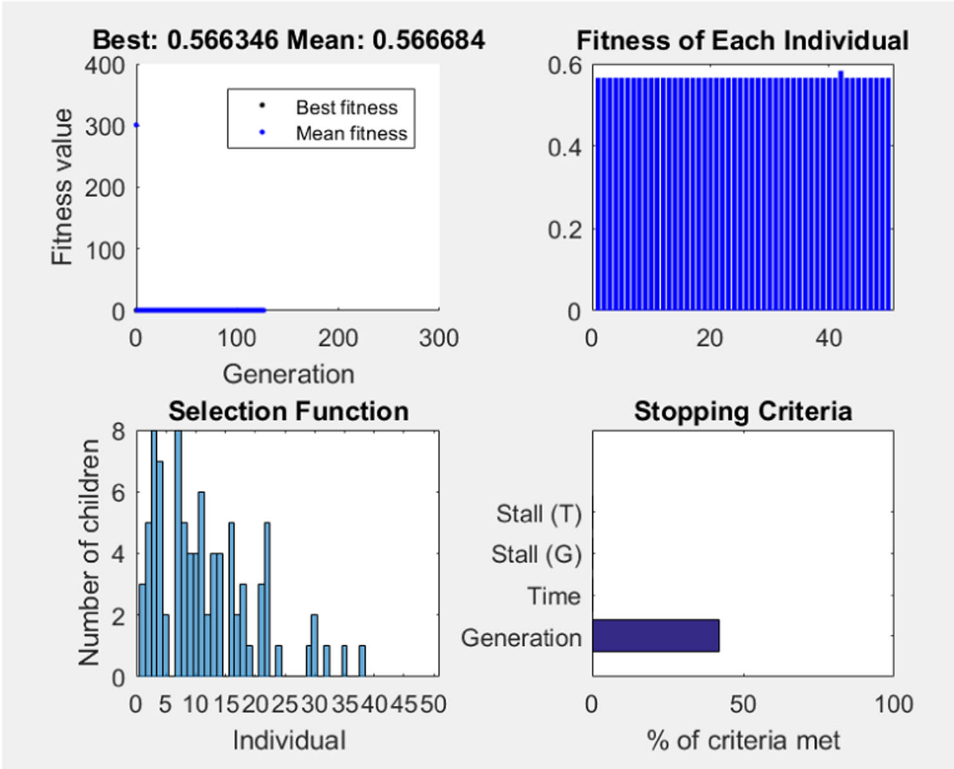

FIGURE 15 Genetic algorithm results. [Color figure can be viewed at wileyonlinelibrary.com]

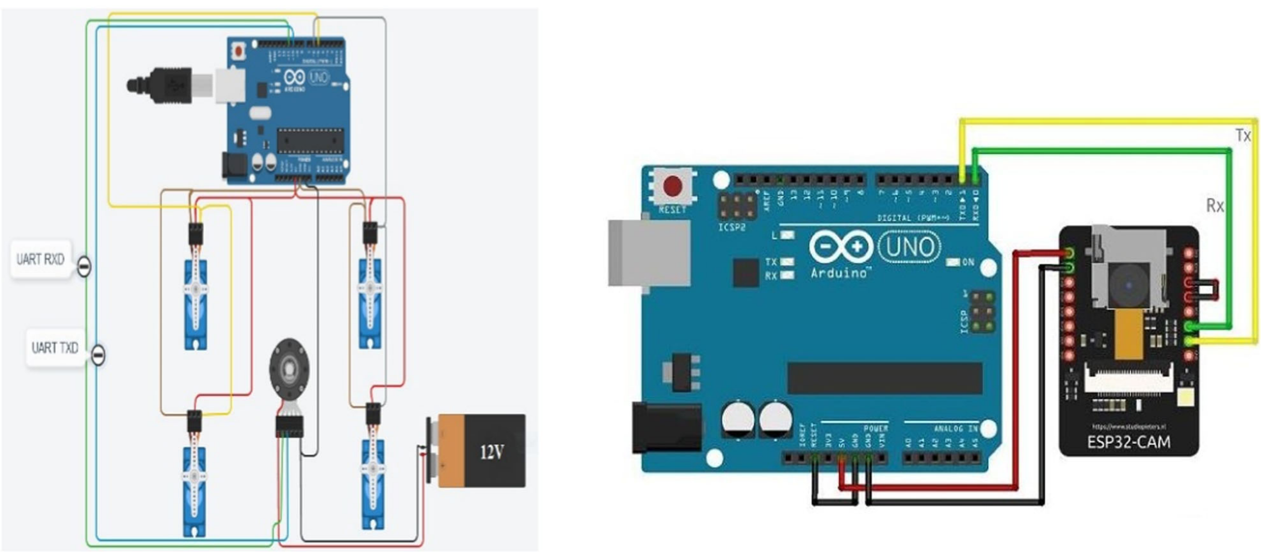

FIGURE 16 Electrical circuit and camera connection diagram. [Color figure can be viewed at wileyonlinelibrary.com]

methodology works well with the physical trials. However, there are certain practical limitations for implementations for the telecom tower maintenance work. The first limitation is the adoptability by the telecom operators and then field staff training requirements for using robots. Another challenge is that environmental conditions like wind and rain may not be favourable for the robot components to work and perform. The costs involved and the economic viability for implementation are the points of concern. For remote telecom sites, managing power and battery life for the performance of robotics systems may be challenging. The proposed design of the robot is capable to climb all three varieties of ladders, but in case of a missing or broken rung, the robot cannot get support of the rung and not possible to advance the motion.

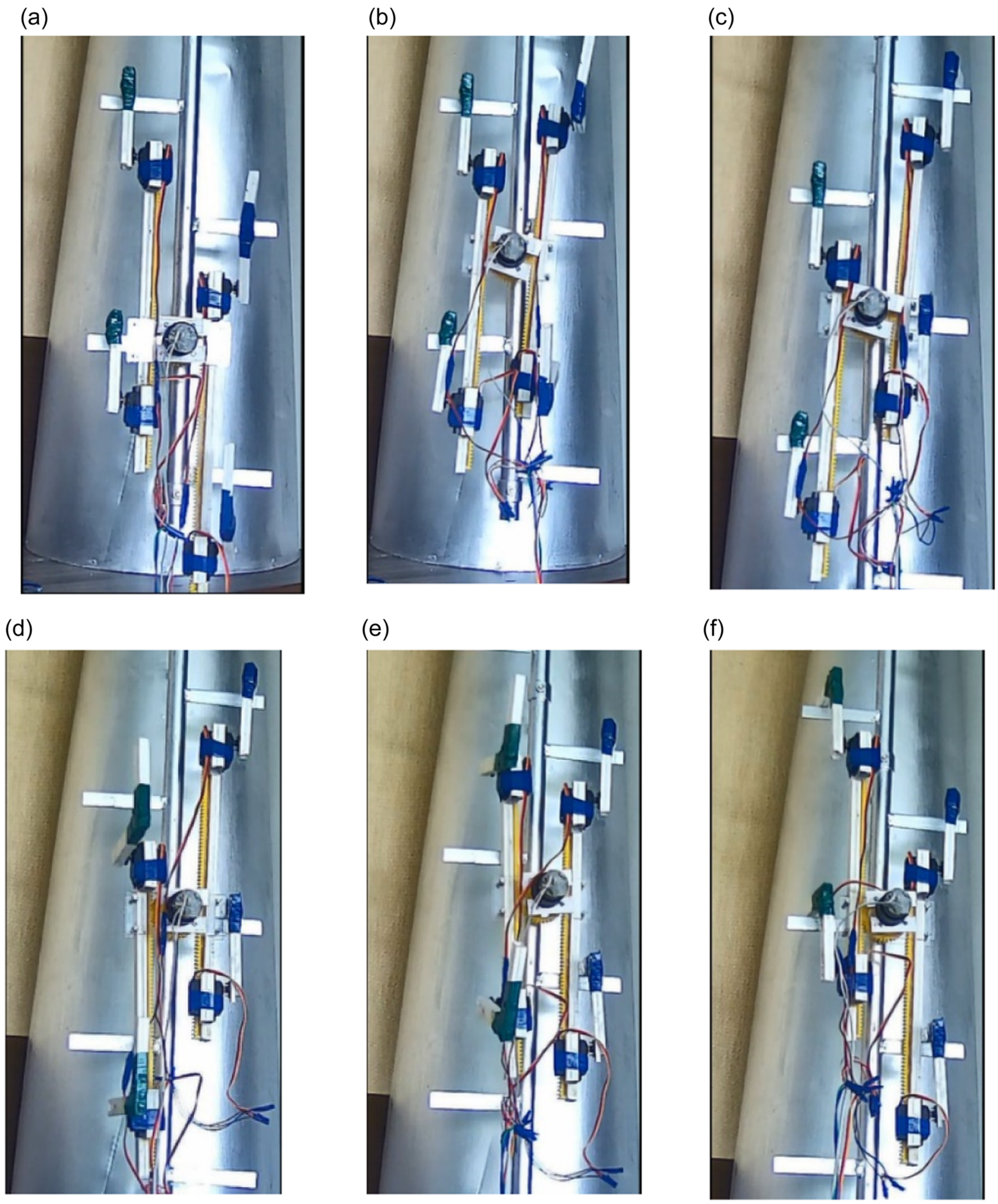

FIGURE 17 Robot climbing experiment. (a) Right clamps open, (b) right rack advances, (c) right clamps close at the next rung, (d) left clamps open, (e) the left rack advances and (f) left clamps close at the next rung. [Color figure can be viewed at wileyonlinelibrary.com]

FIGURE 18 Pictures captured by camera on the top of the tower. [Color figure can be viewed at wileyonlinelibrary.com]

6 | CONCLUSION

The telecommunication sector is growing, and the demand for communication payloads is increasing. It increases the need for many new towers and frequent maintenance of existing network towers. It leads to higher work-at-height hazards and compromises the safety of the field staff. Automation through robotics in the maintenance of telecom towers is a technologically viable option that can reduce the work-at-height hazards. The authors proposed the design, analysis and optimization of a ladder-climbing robot that can climb up the ladder attached to a telecom tower. The static and dynamic analysis along with the design and controlling parameters optimization is presented for the proposed robot.

Design parameters are Optimized using GA for two contradictory objectives, minimization of weight and natural frequency of the robot. A thorough dynamic analysis is conducted, and transfer functions are developed to maintain the dynamic stability of the two-point contact gait system. Transfer functions are simulated using the PID controller in MATLAB Simulink. Results obtained with the GA show the robot's overshoot-free smooth motion. The robot prototype is developed using the arrived PID parameters values and optimized design parameters, and the experimental trial is conducted. The experimental results of a developed prototype on the tower give satisfactory climbing motion. A minor modification in a climbing algorithm is required to climb another type of ladder. The inspection camera attached to a robot took the picture on the top of the tower. That can work as primary information for the operator.

The future extensions for the proposed work can bring solutions to many other societal problems. The findings contribute to advancing ladder-climbing robot technology, it can be extended for further promoting the safe and effective maintenance of telecom towers and other similar infrastructures like towers of electricity, light masts and tall structures like statues and monuments. The solutions can be developed for climbing on uneven or damaged vertical ladders. Mechanisms for applications, such as repairing or replacing tower equipment, performing painting and so forth can be designed and developed. That can be mounted on the climbing robot to perform tasks. The optimization scope can be expanded to increase climbing speed and improve the robot's efficiency. Artificial Intelligence and Machine Learning can be implemented for fault detection by training models with Faulty and unfaulty results.

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SUPPORTING INFORMATION

Additional supporting information can be found online in the Supporting Information section at the end of this article.

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APPENDIX 1

X1	X2	X3	X4	Fx1	Fx2
1.00E−02	9.00E−03	3.00E−01	4.00E−02	2.33E + 00	1.32E + 01
1.50E−02	9.00E−03	4.00E−01	6.00E−02	3.13E + 00	1.14E + 01
1.50E−02	1.00E−02	4.00E−01	6.00E−02	3.13E + 00	1.14E + 01
1.50E−02	1.00E−02	3.00E−01	4.00E−02	2.33E + 00	1.32E + 01
1.10E−02	1.10E−02	3.21E−01	5.32E−02	2.51E + 00	1.27E + 01
1.10E−02	1.10E−02	3.24E−01	5.41E−02	2.53E + 00	1.26E + 01
1.10E−02	1.20E−02	3.86E−01	4.00E−02	3.00E + 00	1.16E + 01
1.20E−02	1.20E−02	3.35E−01	4.19E−02	2.60E + 00	1.25E + 01
2.10E−02	1.20E−02	3.20E−01	5.00E−02	2.69E + 00	1.23E + 01
2.40E−02	1.20E−02	3.87E−01	5.99E−02	3.02E + 00	1.16E + 01
2.80E−02	1.30E−02	3.18E−01	5.89E−02	2.49E + 00	1.27E + 01
2.80E−02	1.30E−02	3.32E−01	4.21E−02	2.58E + 00	1.25E + 01
2.80E−02	1.30E−02	3.64E−01	5.30E−02	2.84E + 00	1.19E + 01
2.80E−02	1.30E−02	3.41E−01	5.86E−02	2.67E + 00	1.23E + 01
3.00E−02	1.40E−02	3.65E−01	5.31E−02	2.85E + 00	1.19E + 01
3.00E−02	1.40E−02	3.72E−01	6.00E−02	2.91E + 00	1.18E + 01
3.00E−02	1.40E−02	3.51E−01	4.06E−02	2.73E + 00	1.22E + 01
3.00E−02	1.40E−02	3.10E−01	5.42E−02	2.43E + 00	1.29E + 01
3.20E−02	1.40E−02	3.47E−01	5.93E−02	2.72E + 00	1.22E + 01
3.20E−02	1.40E−02	3.62E−01	4.00E−02	2.81E + 00	1.20E + 01
3.20E−02	1.40E−02	3.54E−01	4.10E−02	2.74E + 00	1.21E + 01
3.40E−02	1.40E−02	3.40E−01	5.85E−02	2.66E + 00	1.23E + 01
3.40E−02	1.40E−02	3.38E−01	5.91E−02	2.65E + 00	1.24E + 01
3.40E−02	1.40E−02	3.55E−01	4.25E−02	2.75E + 00	1.21E + 01
3.40E−02	1.40E−02	3.28E−01	5.38E−02	2.56E + 00	1.26E + 01
3.60E−02	1.40E−02	3.46E−01	6.00E−02	2.71E + 00	1.22E + 01
3.60E−02	1.40E−02	3.67E−01	5.29E−02	2.86E + 00	1.19E + 01
3.60E−02	1.40E−02	3.70E−01	6.00E−02	2.90E + 00	1.18E + 01
3.60E−02	1.40E−02	3.88E−01	5.91E−02	3.03E + 00	1.16E + 01
3.80E−02	1.40E−02	3.14E−01	4.00E−02	2.44E + 00	1.29E + 01
3.80E−02	1.40E−02	3.14E−01	5.33E−02	2.46E + 00	1.28E + 01
3.80E−02	1.40E−02	3.56E−01	4.15E−02	2.76E + 00	1.21E + 01
3.80E−02	1.40E−02	3.37E−01	5.91E−02	2.64E + 00	1.24E + 01
3.80E−02	1.40E−02	3.05E−01	5.16E−02	2.39E + 00	1.30E + 01
3.80E−02	1.40E−02	3.94E−01	6.00E−02	3.08E + 00	1.15E + 01
4.00E−02	1.50E−02	3.15E−01	4.00E−02	2.45E + 00	1.29E + 01
4.00E−02	1.50E−02	3.62E−01	5.30E−02	2.82E + 00	1.20E + 01
4.00E−02	1.50E−02	3.70E−01	4.00E−02	2.87E + 00	1.19E + 01
4.00E−02	1.50E−02	3.10E−01	5.27E−02	2.42E + 00	1.29E + 01
4.00E−02	1.50E−02	3.79E−01	5.87E−02	2.97E + 00	1.17E + 01
4.00E−02	1.50E−02	3.06E−01	5.26E−02	2.40E + 00	1.30E + 01
1.50E−02	9.00E−03	3.91E−01	6.00E−02	3.06E + 00	1.15E + 01
1.50E−02	1.00E−02	3.62E−01	5.29E−02	2.82E + 00	1.20E + 01
1.50E−02	1.00E−02	3.68E−01	5.88E−02	2.88E + 00	1.19E + 01
1.50E−02	1.10E−02	3.75E−01	6.00E−02	2.93E + 00	1.18E + 01
1.10E−02	1.10E−02	3.75E−01	5.92E−02	2.94E + 00	1.17E + 01
1.10E−02	1.20E−02	3.56E−01	5.21E−02	2.78E + 00	1.21E + 01
1.10E−02	1.20E−02	3.41E−01	5.81E−02	2.67E + 00	1.23E + 01
1.20E−02	1.20E−02	3.25E−01	6.00E−02	2.55E + 00	1.26E + 01
2.10E−02	1.20E−02	3.36E−01	5.93E−02	2.63E + 00	1.24E + 01
2.40E−02	1.30E−02	3.73E−01	6.00E−02	2.92E + 00	1.18E + 01
2.80E−02	1.30E−02	3.66E−01	5.88E−02	2.86E + 00	1.19E + 01
2.80E−02	1.30E−02	3.16E−01	5.33E−02	2.47E + 00	1.28E + 01
2.80E−02	1.30E−02	3.78E−01	5.96E−02	2.96E + 00	1.17E + 01
2.80E−02	1.40E−02	3.45E−01	6.00E−02	2.70E + 00	1.22E + 01
3.00E−02	1.40E−02	3.36E−01	4.10E−02	2.61E + 00	1.25E + 01
3.00E−02	1.40E−02	3.76E−01	5.95E−02	2.94E + 00	1.17E + 01
3.00E−02	1.40E−02	3.00E−01	5.12E−02	2.34E + 00	1.31E + 01
3.00E−02	1.40E−02	3.27E−01	5.38E−02	2.56E + 00	1.26E + 01
3.20E−02	1.40E−02	3.77E−01	5.98E−02	2.95E + 00	1.17E + 01
3.20E−02	1.40E−02	3.81E−01	5.87E−02	2.98E + 00	1.17E + 01
3.20E−02	1.40E−02	3.03E−01	4.00E−02	2.35E + 00	1.31E + 01
3.40E−02	1.40E−02	3.07E−01	4.00E−02	2.38E + 00	1.30E + 01
3.40E−02	1.40E−02	3.57E−01	5.21E−02	2.79E + 00	1.21E + 01
3.40E−02	1.40E−02	3.44E−01	5.99E−02	2.70E + 00	1.23E + 01
3.40E−02	1.40E−02	3.04E−01	5.28E−02	2.37E + 00	1.31E + 01
3.60E−02	1.40E−02	3.15E−01	4.00E−02	2.44E + 00	1.29E + 01
3.60E−02	1.40E−02	3.99E−01	4.20E−02	3.09E + 00	1.14E + 01
3.60E−02	1.40E−02	3.02E−01	5.12E−02	2.36E + 00	1.31E + 01
3.60E−02	1.40E−02	3.82E−01	6.00E−02	2.99E + 00	1.16E + 01
3.80E−02	1.40E−02	3.55E−01	5.22E−02	2.77E + 00	1.21E + 01
3.80E−02	1.40E−02	3.36E−01	5.95E−02	2.63E + 00	1.24E + 01
3.80E−02	1.40E−02	3.31E−01	4.26E−02	2.57E + 00	1.25E + 01
3.80E−02	1.40E−02	3.93E−01	6.00E−02	3.08E + 00	1.15E + 01
3.80E−02	1.40E−02	3.02E−01	5.17E−02	2.36E + 00	1.31E + 01
3.80E−02	1.50E−02	3.03E−01	4.09E−02	2.35E + 00	1.31E + 01
4.00E−02	1.50E−02	3.53E−01	4.10E−02	2.74E + 00	1.22E + 01
4.00E−02	1.50E−02	3.60E−01	5.22E−02	2.81E + 00	1.20E + 01
4.00E−02	1.50E−02	3.78E−01	5.88E−02	2.96E + 00	1.17E + 01
4.00E−02	1.50E−02	3.01E−01	4.00E−02	2.34E + 00	1.32E + 01
4.00E−02	1.50E−02	3.49E−01	5.98E−02	2.73E + 00	1.22E + 01
4.00E−02	9.00E−03	3.89E−01	5.94E−02	3.04E + 00	1.15E + 01
1.50E−02	1.00E−02	3.18E−01	5.34E−02	2.48E + 00	1.28E + 01
1.50E−02	1.00E−02	3.97E−01	6.00E−02	3.11E + 00	1.14E + 01
1.50E−02	1.10E−02	3.17E−01	5.89E−02	2.48E + 00	1.28E + 01
1.50E−02	1.10E−02	4.00E−01	4.28E−02	3.10E + 00	1.14E + 01
1.10E−02	1.20E−02	3.56E−01	5.33E−02	2.77E + 00	1.21E + 01
1.10E−02	1.20E−02	3.99E−01	4.07E−02	3.09E + 00	1.14E + 01
1.10E−02	1.20E−02	3.58E−01	5.23E−02	2.79E + 00	1.20E + 01
1.20E−02	1.20E−02	3.24E−01	5.96E−02	2.54E + 00	1.26E + 01
2.10E−02	1.30E−02	3.15E−01	5.96E−02	2.48E + 00	1.28E + 01
2.40E−02	1.30E−02	3.01E−01	4.00E−02	2.34E + 00	1.32E + 01
2.80E−02	1.30E−02	3.73E−01	6.00E−02	2.92E + 00	1.18E + 01
2.80E−02	1.30E−02	3.15E−01	5.33E−02	2.46E + 00	1.28E + 01
2.80E−02	1.40E−02	3.07E−01	5.26E−02	2.40E + 00	1.30E + 01
2.80E−02	1.40E−02	3.59E−01	5.37E−02	2.80E + 00	1.20E + 01
3.00E−02	1.40E−02	3.58E−01	5.23E−02	2.80E + 00	1.20E + 01
3.00E−02	1.40E−02	3.03E−01	5.27E−02	2.37E + 00	1.31E + 01
3.00E−02	1.40E−02	3.87E−01	5.99E−02	3.02E + 00	1.16E + 01
3.00E−02	1.40E−02	3.92E−01	6.00E−02	3.07E + 00	1.15E + 01

APPENDIX 2

Generation	Func-count	Best *f*(*x*)	Mean *f*(*x*)	Stall generations
1	100	1.93E−31	0.4464	0
2	147	1.93E−31	0.2332	1
3	194	1.93E−31	0.1695	2
4	241	1.93E−31	0.155	3
5	288	4.88E−32	0.1571	0
6	335	3.49E−32	0.09191	0

Generation	Func-count	Best f(x)	Mean f(x)	Stall generations
7	382	3.49E−32	3.71E−05	1
8	429	2.94E−32	0.01907	0
9	476	2.79E−33	2.02E−05	0
10	523	2.79E−33	2.04E−05	1
11	570	1.60E−33	9.18E−06	0
12	617	1.60E−33	1.11E−05	1
13	664	1.60E−33	6.40E−06	2
14	711	1.60E−33	5.98E−06	3
15	758	1.60E−33	1.53E−06	4
16	805	1.60E−33	1.03E−06	5
17	852	1.60E−33	8.93E−07	6
18	899	6.66E−34	7.26E−07	0
19	946	6.66E−34	1.21E−07	1
20	993	6.66E−34	4.75E−08	2
21	1040	6.66E−34	2.91E−08	3
22	1087	1.96E−36	1.55E−08	0
23	1134	1.96E−36	2.10E−08	1
24	1181	1.96E−36	2.01E−08	2
25	1228	1.96E−36	9.57E−09	3
26	1275	1.96E−36	1.03E−08	4
27	1322	1.96E−36	5.26E−09	5
28	1369	1.96E−36	3.26E−09	6

(Continues)

Generation	Func-count	Best f(x)	Mean f(x)	Stall generations
29	1416	1.96E−36	1.64E−09	7
30	1463	1.96E−36	2.10E−08	8
31	1510	1.96E−36	2.06E−08	9
32	1557	1.96E−36	2.05E−08	10
33	1604	1.96E−36	4.40E−10	11
34	1651	1.96E−36	3.16E−10	12
35	1698	1.11E−36	2.02E−08	0
36	1745	1.11E−36	4.90E−10	1
37	1792	1.11E−36	2.83E−10	2
38	1839	1.11E−36	2.01E−08	3
39	1886	1.11E−36	1.57E−10	4
40	1933	1.11E−36	2.40E−10	5
41	1980	1.11E−36	2.01E−08	6
42	2027	1.11E−36	2.95E−10	7
43	2074	1.11E−36	2.78E−10	8
44	2121	1.11E−36	3.77E−10	9
45	2168	1.11E−36	2.72E−10	10
46	2215	1.11E−36	5.11E−10	11
47	2262	1.11E−36	2.63E−10	12
48	2309	1.11E−36	5.38E−10	13
49	2356	1.11E−36	5.41E−10	14
50	2403	1.11E−36	3.61E−10	15